MATH 2153 - Calculus III – Recitation 1

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§12.1-12.2: Vectors in the plane and in space.

1. Let $P = (-2, 3)$ and $Q = (-4, 10)$.
   (a) Compute $|\overrightarrow{PQ}|$.
   (b) Compute $|\overrightarrow{QP}|$.
   (c) Is it true that for any points R and S in the plane
   $$|\overrightarrow{RS}| = |\overrightarrow{SR}|?$$
   Prove this or give a counterexample.

2. Let $\mathbf{i} = \langle 1, 0\rangle$ and $\mathbf{j} = \langle 0, 1\rangle$ be the **coordinate unit vectors** in the plane and let $\mathbf{u} = \langle 1, 1\rangle$.
   (a) Write the vector $\langle -3, 4\rangle$ in the form $a\mathbf{i} + b\mathbf{j}$ where a and b are scalars.
   (b) Describe the set of vectors which can be written in the form $a\mathbf{i}$ where a is a scalar.
   (c) Write the vector $\langle -3, 4\rangle$ in the form $a\,\mathbf{i} + b\,\mathbf{j} + c\,\mathbf{u}$ where a, b and c are scalars in two different ways.
   (d) $\langle -3, 4\rangle$ in the form $a\,\mathbf{i} + c\,\mathbf{u}$ where a and c are scalars.

3. **Boat in a wind.** A sailboat floats in a current that flows due east at 1 m/s. Due to a wind, the boat's actual speed relative to the shore is $\sqrt{3}$ m/s in a direction 30° northeast. Find the speed and direction of the wind. (*Hint:* Draw a picture.)

4. A **unit vector** in space is a vector with magnitude 1.
   (a) Give a unit vector which makes an angle of $\pi/4$ with the z-axis and lies in one of the three coordinate planes through the origin. How many can you find?
   (b) Compute $|\langle 1, 2, 4\rangle|$ and $|5\langle 1, 2, 4\rangle|$.
   (c) Give a unit vector which is parallel to the vector $\langle 1, 2, 4\rangle$. How many can you find?

5. (a) Find an equation or inequality that describes a sphere with center $(1, 2, 3)$ and radius 10.
   (b) Give a geometric description of the set of points satisfying $x^2 + y^2 - 14y + z^2 \geq -13$.

6. **Crosswinds.** A small plane is flying horizontally due east in calm air at 250 mi/hr when it is hit by a horizontal crosswind blowing southwest at 50 mi/hr and a 30 mi/hr updraft. Find the resulting speed of the plane and describe with a sketch the approximate direction of the velocity vector relative to the ground.