Recitation I (1/14/16)

Problem 1: (a) $|\overrightarrow{PQ}| = \sqrt{(-4-(-2))^2 + (10-3)^2} = \sqrt{4+49} = \sqrt{53}$

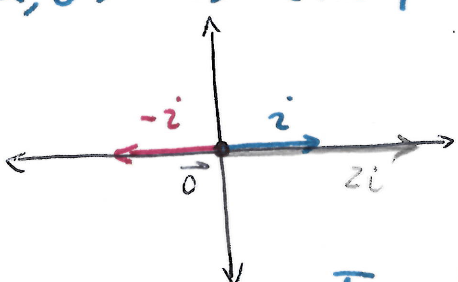
(b) $\overrightarrow{QP} = -\overrightarrow{PQ} \implies |\overrightarrow{QP}| = \sqrt{53}$

(c) TRUE. Reason: $\overrightarrow{RS} = -\overrightarrow{SR} \implies |\overrightarrow{RS}| = |-1|\,|\overrightarrow{SR}| = |\overrightarrow{SR}|$.

Problem 2: (a) $\langle -3, 4\rangle = \langle -3, 0\rangle + \langle 0, 4\rangle = \underbrace{-3}\, i + \underbrace{4}\, j$

This expression is UNIQUE! $\left(ai + bj = \langle a, b\rangle \implies \begin{matrix} a=-3 \\ b=4 \end{matrix} \right)$

(b) $ai = \langle a, 0\rangle \implies$ Examples: i, $2i$, $\vec{0}$, $-i$



Answer: We get any position vector whose head lies in the x-axis.

$= \{ \langle a, 0\rangle$: a in $\mathbb{R} \}$

(c) $\langle -3, 4\rangle$ can be written in infinitely many ways as a linear combination of $i, j, \underline{u}$.

Examples: $\langle -3, 4\rangle = -3i + 4j + 0\underline{u}$
$= -7i + 0j + 4\underline{u}$
$= 0i + (-7)j + 4\underline{u}$
$= -4i + 3j + \underline{u}$

In general: $ai + bj + c\underline{u} = ai + bj + c(i+j) = (a+c)i + (b+c)j$

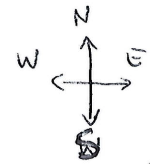
$$\implies \begin{cases} -3 = a + c \\ 4 = b + c \end{cases}$$

(d) $\langle -3, 4\rangle = \underbrace{-7}\, i + \underbrace{4}\, \underline{u}$

In this case, the expression is UNIQUE! $(ai + c\underline{u} = \langle a+c, c\rangle$

so $a + c = -3 \implies a = -7$
$c = 4$

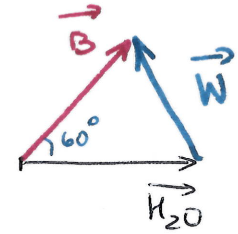
Problem 3: We start by sketching the problem. There are 3 vectors playing a role:



$\vec{B}$ = boat velocity

$\vec{W}$ = wind velocity

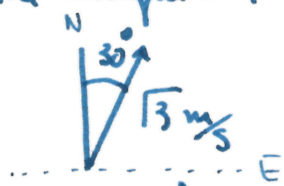
$\overrightarrow{H_2O}$ = vector describing the water flow $= \langle 1, 0 \rangle$



We know:

$$\vec{B} = \vec{W} + \overrightarrow{H_2O}$$

To determine $\vec{B}$, we use the magnitud and angle information.



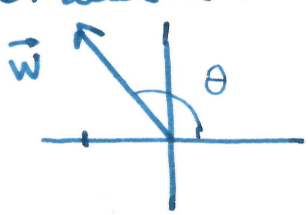
$$\vec{B} = \sqrt{3}\,\langle \operatorname{sen} 30°, \cos 30° \rangle = \sqrt{3}\,\langle \tfrac{1}{2}, \tfrac{\sqrt{3}}{2} \rangle = \langle \tfrac{\sqrt{3}}{2}, \tfrac{3}{2} \rangle$$

$$\Rightarrow \vec{W} = \vec{B} - \overrightarrow{H_2O} = \langle \frac{\sqrt{3}-2}{2}, \frac{3}{2} \rangle$$

The coordinates have sign $(-,+)$
$\Rightarrow$ wind blows Northwest.

$$\text{Speed} = |\vec{W}| = \frac{1}{2}\sqrt{(\sqrt{3}-2)^2 + 9} = \frac{\sqrt{22-4\sqrt{3}}}{2} \approx 3.88.$$

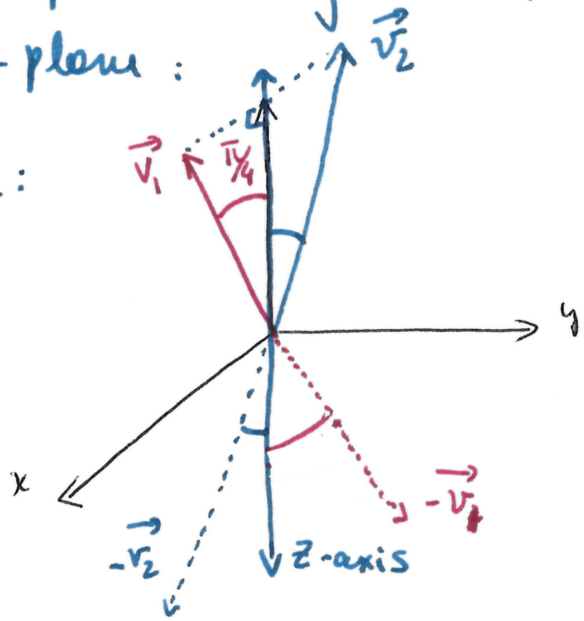
We can determine the angle:



$$\cos\theta = \frac{\sqrt{3}-2}{\sqrt{22-4\sqrt{3}}} \qquad \Rightarrow \theta \approx 40° \text{ NW.}$$

$$\operatorname{sen}\theta = \frac{3}{\sqrt{22-4\sqrt{3}}}$$

Problem 4: (a) We can find exactly 4 in the ~~x~~z-plane and 4 in the yz-plane:



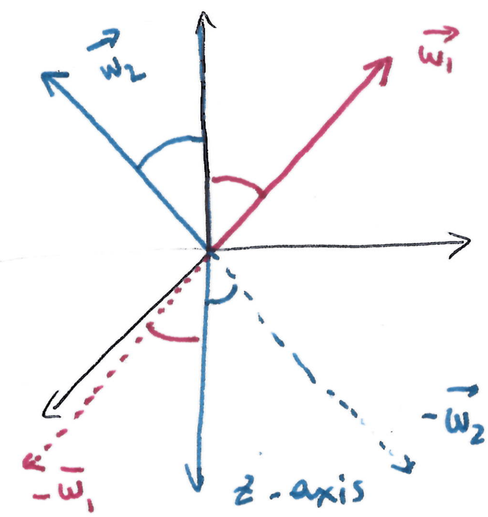
- For xz-plane:

$$\vec{v_1} = 1\left< \sin\frac{\pi}{4}, 0, \cos\frac{\pi}{4} \right> = \left< \frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2} \right>$$

$$\vec{v_2} = 1\left< -\sin\frac{\pi}{4}, 0, \cos\frac{\pi}{4} \right> = \left< -\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2} \right>$$

Other 2 solutions: $-\vec{v_2}$ & $-\vec{v_1}$.

- For yz-plane:



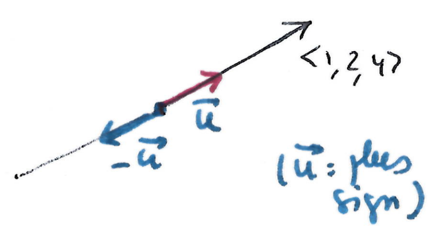
$$\vec{w_1} = 1\left< 0, \sin\frac{\pi}{4}, \cos\frac{\pi}{4} \right> = \left< 0, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right>$$

$$\vec{w_2} = 1\left< 0, -\sin\frac{\pi}{4}, \cos\frac{\pi}{4} \right> = \left< 0, -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right>$$

Other 2 solutions: $-\vec{w_1}$ & $-\vec{w_2}$.

(b) $|\langle 1,2,4\rangle| = \sqrt{1^2+2^2+4^2} = \sqrt{21}$

$|5\langle 1,2,4\rangle| = |5|\sqrt{21} = 5\sqrt{21}$.



(c) 2 solutions: $\pm\left< \frac{1}{\sqrt{21}}, \frac{2}{\sqrt{21}}, \frac{4}{\sqrt{21}} \right>$

Problem 5: (a) $(x-1)^2+(y-2)^2+(z-3)^2=100.$

(b) We complete squares in the y-variable:

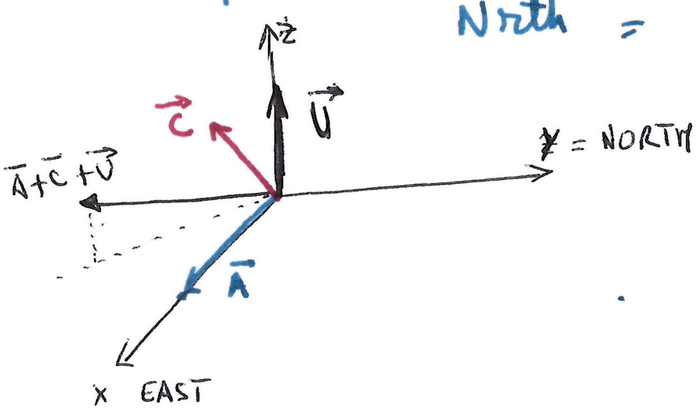
$$x^2+y^2-14y+z^2 = x^2+(y-7)^2-49+z^2 \geq -13$$

$$\boxed{x^2+(y-7)^2+z^2 \geq 36 = 6^2}$$

The solution set consist of all points lying <u>OUTSIDE</u> the interior of the ball with center $(0,7,0)$ & radius 6, that is, the complement of the ball & also the points in the sphere.

Problem 6: As with Problem 3, we sketch the situation.

- Our xy-plane is the horizontal plane containing the plane.
- For reference: East = positive x-axis
  North = " y-axis



Airplane velocity $= \vec{A} = 250\,\mathbf{i}$

Crosswind $= \vec{C} = 50\dfrac{\langle -1,-1,0\rangle}{\sqrt{2}}$

$= 25\langle -\sqrt{2}, -\sqrt{2}, 0\rangle$

$= -25\sqrt{2}\,\mathbf{i} - 25\sqrt{2}\,\mathbf{j}$

Updraft $= \vec{U} = 30\langle 0,0,1\rangle$

$= 30\,\vec{k}.$

Direction SW = neg. x & neg y coord

Angle $\frac{\pi}{4}$ $\Rightarrow$ x & y coord have the same value.

$\Rightarrow$ New velocity of plane $= \vec{A}+\vec{C}+\vec{V} = \langle \overbrace{250-25\sqrt{2}}^{\text{positive}}, \overbrace{-25\sqrt{2}}^{\text{neg.}}, 30\rangle$

$\Rightarrow$ New speed $= \sqrt{(250-25\sqrt{2})^2+(-25\sqrt{2})^2+30^2} \approx 219.596$ mi/h.