Recitation II (1/21/16)

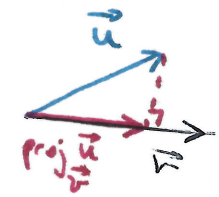
Problem 1: Use geometric definition of the dot product

$\vec{u}\cdot\vec{v} = |\vec{u}|\,|\vec{v}|\cos\theta \qquad 0 \le \theta \le \pi$ angle between $\vec{u}$ & $\vec{v}$

So: $1 = \langle 1,1\rangle \cdot \langle 1,0\rangle = \sqrt{2}\cdot 1 \cos\theta \Rightarrow \cos\theta = \frac{\sqrt{2}}{2} \Rightarrow \boxed{\theta = 45^\circ = \frac{\pi}{4}}$

Problem 2: Recall that

$$\begin{cases} \text{proj}_{\vec{v}}\vec{u} = \text{comp}_{\vec{v}}\vec{u}\,\dfrac{\vec{v}}{|\vec{v}|} \\ \text{comp}_{\vec{v}}\vec{u} = \dfrac{u\cdot v}{|\vec{v}|} \end{cases}$$



(a) So $\text{proj}_{\vec{z}}\vec{u} = \frac{3}{1}\,\frac{\vec{z}}{1} = \langle 3,0,0\rangle$

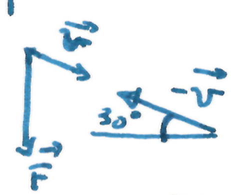
$\text{comp}_{\vec{z}}\vec{u} = 3$

(b) $\text{proj}_{\vec{k}} u = \left(\text{comp}_{\vec{k}}\vec{u}\right)\vec{k} = \frac{u_3}{1}\,\vec{k} = \langle 0,0,u_3\rangle$

Problem 3: $\vec{F} = 10\langle 0,-1\rangle$

(a) We need to find the direction $\vec{v}$ parallel to the plane and then compute $|\text{proj}_{\vec{v}}\vec{F}| = |\text{comp}_{\vec{v}}\vec{F}| = \frac{|\vec{F}\cdot\vec{v}|}{|\vec{v}|}$.

$-\vec{v}$ forms an angle of 30° with $\langle -1,0\rangle$

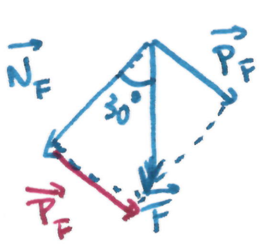


We conclude $-\vec{v} = \langle -\cos 30^\circ, \sin 30^\circ\rangle$

$-\vec{v} = \langle -\frac{\sqrt{3}}{2}, \frac{1}{2}\rangle$, so $\vec{v} = \langle +\frac{\sqrt{3}}{2}, \frac{-1}{2}\rangle$

$\vec{F}\cdot\vec{v} = \frac{+10}{2} = 5$ gives $|\text{proj}_{\vec{v}}\vec{F}| = \boxed{5}$

Alternative solution:



so $|\vec{P}_F| = |\vec{F}|\cdot\sin 30^\circ = 5$.

(b) Claim: normal component + parallel component = $\vec{F}$.

Parallel comp $= \text{proj}_{\vec{v}} \vec{F} = \dfrac{\vec{F}\cdot\vec{v}}{|\vec{v}|}\dfrac{\vec{v}}{|\vec{v}|} = 5\left\langle \dfrac{\sqrt{3}}{2}, \dfrac{-1}{2}\right\rangle$

$= \left\langle \dfrac{5\sqrt{3}}{2}, \dfrac{-5}{2}\right\rangle$

So $\vec{F} - \left\langle \dfrac{5\sqrt{3}}{2}, \dfrac{-5}{2}\right\rangle = \left\langle \dfrac{-5\sqrt{3}}{2}, -10+\dfrac{5}{2}\right\rangle = \left\langle \dfrac{-5\sqrt{3}}{2}, \dfrac{-15}{2}\right\rangle$ is the normal comp.

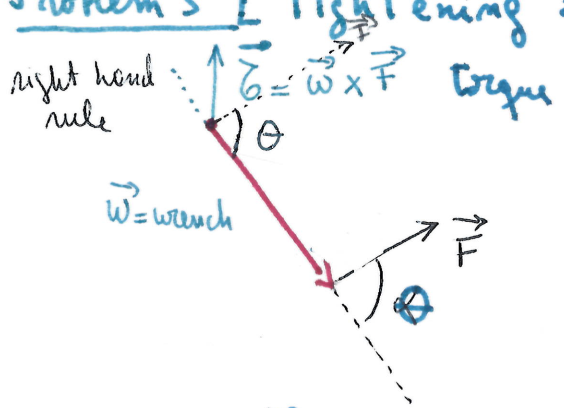
Sanity check: $\vec{N}_F \cdot \vec{P}_F = 0$. ✓

Problem 4: (a) Use dot product:

$$0 \stackrel{?}{=} \langle 3, b, 2b\rangle \cdot \langle 2, 3, -1\rangle = 6 + 3b - 2b = 6 + b \Rightarrow \boxed{b = -6}$$

(b) $0 \stackrel{?}{=} \langle 1, q\rangle \cdot \langle 1, 0\rangle = 1$ for all q, so we cannot find any q so that $\langle 1, q\rangle$ is orthogonal to $\langle 1, 0\rangle$.

Problem 5 [Tightening a bolt]



Situation 1: $|\vec{w}| = 0.1$
$\theta = 60°$

Situation 2: $|\vec{w}| = 0.25$
$\theta = 135°$

Force $= |\vec{F}| = 40$

We want to find $|\vec{\tau}| = |\vec{w} \times \vec{F}| = |\vec{w}|\,|\vec{F}| \sin\theta$

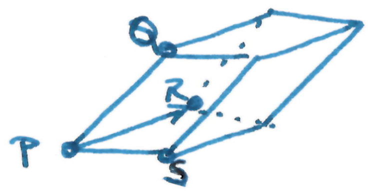
Situation 1: $|\vec{\tau}| = 0.1\,\text{m} \cdot 40\,\text{N} \cdot \sin 60° = 0.1\,\text{m}\, 40\,\text{N} \dfrac{\sqrt{3}}{2} = 2\sqrt{3}\ \text{N m} = 2\sqrt{3}\ \text{J}$

Situation 2: $|\vec{\tau}| = 0.25\,\text{m} \cdot 40\,\text{N} \sin 135° = 0.25 \cdot 40 \dfrac{\sqrt{2}}{2}\ \text{J} = 5\sqrt{2}\ \text{J}$

So situation 2 produces more torque than Situation 1 because $5\sqrt{2} > 2\sqrt{3}$

The direction of the Torque is in the positive z-axis (using the right hand rule).

Problem 6: To determine the points are coplanar it suffices to show that the parallelopide with corners at the 4 points has volume equal to 0.



(sketch)

$P = (1,0,0)$
$Q = (0,1,1)$
$R = (1,0,2)$
$S = (5,-4,-2)$

$\vec{PQ} = \langle -1, 1, 1 \rangle$

$\vec{PR} = \langle 0, 0, 2 \rangle$

$\vec{PS} = \langle 4, -4, -2 \rangle$

$\Rightarrow \text{Vol} = |\vec{PQ} \cdot (\vec{PR} \times \vec{PS})|$

$$\vec{PR} \times \vec{PS} = \begin{vmatrix} i & j & k \\ 0 & 0 & 2 \\ 4 & -4 & -2 \end{vmatrix} = 8\vec{i} - \vec{j}(-8) + \vec{k}\,0 = \langle 8, 8, 0 \rangle$$

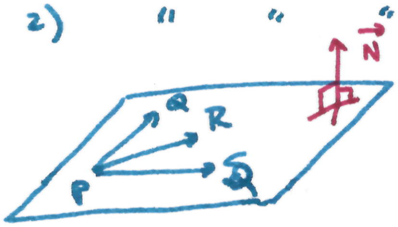
$$\vec{PQ} \cdot (\vec{PR} \times \vec{PS}) = \langle -1, 1, 1 \rangle \cdot \langle 8, 8, 0 \rangle = -8 + 8 + 0 = \boxed{0}$$

Alternative Solution:

We'll see later that a plane is defined in 2 ways

1) contains a point P and a normal direction $\vec{N}$ ($\vec{N}$ is perpendicular to any direction in the plane)

2) " " " " has 2 directions



To find one such $\vec{N}$ we use cross product:

$\vec{N} = \vec{PR} \times \vec{PS} = \langle 8, 8, 0 \rangle$ (done above)

Need to check $\vec{N} \perp \vec{PQ}$: use dot product

$$\langle 8, 8, 0 \rangle \cdot \vec{PQ} = \langle 8, 8, 0 \rangle \cdot \langle -1, 1, 1 \rangle = \boxed{0}$$