MATH 2153 - Calculus III – Recitation 4

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February 04, 2016

Note: Ignore all questions about torsion and TNB-frames. We will see these concepts on Friday.

1. (a) Find the equation of the curve $\mathbf{r}(t)$ satisfying:
   i. the tangent vector at time t equals $\langle \sqrt{2}\,t, 1, -t^2 \rangle$;
   ii. the curve passes through $\langle 1, 1, 1 \rangle$ at time $t = 0$.

   (b) Compute the length of the curve between times $t = 0$ and $t = 6$.

   (c) Compute the curvature of $\mathbf{r}(t)$ at time t using the two formulas given in class, and find the time at which the trajectory is most curved (i.e., when $\kappa(t)$ is largest).

   (d) Compute the torsion of $\mathbf{r}(t)$ at time t, and find the time at which the trajectory is most twisted (i.e., when $\tau(t)$ is largest).

2. Let $\mathbf{r}(t) = \langle 3\sin t, 4\,t, 3\cos t \rangle$ for $0 \leq t \leq 5$.

   (a) Find the position vector of the point at a distance 5 from $\mathbf{r}(0) = \langle 0, 0, 3 \rangle$.

   (b) Find the arc length reparameterization of the curve $\mathbf{r}$.

3. Let $s(t)$ be the arc length function for the curve $\mathbf{r}(t) = \ln t\,\mathbf{i} + t^{-1}\,\mathbf{j} + t\,\mathbf{k}$. Compute $\frac{ds}{dt}$ at $t = 2$.

4. (a) Show that the curvature and torsion of the helix $\mathbf{r}(t) = \langle \cos t, \sin t, 2t \rangle$ are constant. What are their values?

   (b) Write the scalar components of the trajectory, velocity and acceleration for the TNB-frame.

5. Find the tangential and normal components of acceleration for the trajectory $\mathbf{r}(t) = t^3\,\mathbf{i} + t^2\,\mathbf{j}$.