Problem 1: 

\( f(x,y) = x + 2y \) is continuous, so we use Fubini & the Fund. Thm. of Calc., to get \( A(x) \) & \( A_y \):

\[
\int_0^3 \left( \int_0^1 f(x,y) \, dy \right) \, dx = \int_0^3 \left( \int_0^1 (x + 2y) \, dy \right) \, dx = \int_0^3 \left( \left[ xy + y^2 \right]_y^1 \right) \, dx
\]

\[
= \int_0^3 \left( 4x + 16 - (x + 1) \right) \, dx = \int_0^3 3x + 15 \, dx = \frac{3}{2} x^2 + 15 x \bigg|_{x=0}^{x=3} = \frac{27}{2} + 45 = \frac{117}{2}
\]

Check the other order of integration:

\[
\int_0^1 \left( \int_0^3 f(x,y) \, dx \right) \, dy = \int_0^1 \left( \int_0^3 (x + 2y) \, dx \right) \, dy = \int_0^1 \left( \left[ \frac{x^2}{2} + 2xy \right]_x^3 \right) \, dy
\]

\[
= \int_0^1 \left( \frac{9}{2} + 6y - 0 \right) \, dy = \left( \frac{9}{2} y + 3y^2 \right) \bigg|_{y=0}^{y=1} = \frac{36}{2} + 98 - \left( \frac{9}{2} + 3 \right)
\]

\[
= 63 - \frac{9}{2} = \frac{117}{2} \quad \text{as we expected!}
\]

Problem 2: Since \( \frac{\partial f}{\partial y} \) is continuous, we can compute the double integral using Fubini: since the derivatives are notated from right to left we choose to integrate \( y \)-slices:

\[
\iint_R f(x,y) \, dA = \int_0^b \left( \int_0^a \frac{\partial f}{\partial y} \, dx \right) \, dy = \int_0^b \left( \int_0^a \frac{\partial f}{\partial x} \left( \frac{\partial f}{\partial y} \right) \, dx \right) \, dy
\]

\[
= \int_0^b \left. \frac{\partial f}{\partial y} \right|_x^a \, dy = \int_0^b \left( \frac{\partial f}{\partial y}(a,y) - \frac{\partial f}{\partial y}(0,y) \right) \, dy = \left( f(a,y) - f(0,y) \right) \bigg|_0^b
\]

\[
= f(a,b) - f(0,b) - (f(a,0) - f(0,0))
\]

\[
= f(a,b) - f(0,b) - f(a,0) + f(0,0) = \boxed{f(0,0)}
\]

because they are continuous & we are fixing \( x \) different with \( y \).
Notice: we could have integrated the x-slices instead because $f_{xy} = f_{yx}$ since both functions are continuous by assumption.

Problem 3: We start by drawing the tetrahedron:

In this, we draw the intercepts of the plane $z = c - ax - by$

- $x = y = 0 \implies z = c$
- $x = z = 0 \implies y = \frac{c}{b}$
- $y = z = 0 \implies x = \frac{c}{a}$

The volume is obtained by integrating the function $f(x,y) = c - ax - by$ over the triangle $(0,0)$, $(\frac{c}{a},0)$, $(\frac{c}{b},0)$ using Fubini.

We choose to integrate first over $x$: then we need to find the values of $y$. For this we parameterize the 2 segments $(0,0)$ to $(\frac{c}{a},0)$ and $(\frac{c}{b},0)$ to $(\frac{c}{b},0)$ as $y = y(x)$:

1. $y = y(0) = 0 / (0,0)$
2. $y = y(x) = m(x - \frac{c}{a})$

To determine $m$: $x = 0$ since $\frac{c}{b} = m(-\frac{c}{a}) \implies m = \frac{a}{b}$

$$\text{Vol}(\Delta) = \int_0^{\frac{c}{a}} \left( \int_0^{\frac{c}{b}} (c - ax - by) \, dy \right) \, dx$$

$$= \int_0^{\frac{c}{a}} \left( \int_0^{\frac{c}{b}} \left( cy - axy - by \right) \, dy \right) \, dx$$

$$= \int_0^{\frac{c}{a}} \left[ cy - axy - \frac{b}{2}y^2 \right]_0^{\frac{c}{b}} \, dx$$

$$= \int_0^{\frac{c}{a}} \left( cn - ax\frac{c}{b} - \frac{c^2}{2b^2} \right) \, dx$$

$$= \int_0^{\frac{c}{a}} \left( c - ax\frac{c}{b} + \frac{c^2}{2b^2} \right) \, dx$$

$$= \left[ \frac{1}{2} \left( \frac{c^2}{2b} \right)^2 - \frac{1}{2} \left( -ax + c \right)^2 \right]_0^{\frac{c}{a}} \, dx$$

$$= \int_0^{\frac{c}{a}} \frac{c^2}{2b} - \frac{1}{2} \left( -ax + c \right)^2 \, dx$$

$$= \left( \frac{c^2}{2b} \right) \left[ \frac{1}{2} \left( -ax + c \right)^2 \right]_0^{\frac{c}{a}}$$

$$= \left( \frac{c^3}{2b} \right) \left[ -\left( 0^3 - \frac{c^3}{2b} \right) \right] = \frac{c^3}{2b}.$$
Problem 4: We start by drawing the solid. For this we complete the 3 pairwise intersections:

- The red inverted parabola is the parabola on the $z=2+y$ plane.
- The green parabola $z=2+x^2$ is the parabola on the $x^2=0$ plane.
- The blue parabola $z=6-y$.

The base curve can be projected to the $xy$-plane:

- $z=2+x^2$  \( \Rightarrow \)  $6-y=2+x^2$ \[4-x^2=y\]

The volume of the solid is the volume in between the graphs of 2 functions defined over $R$.

1. First (top) graph: $z=f(x,y) = 6-y$
2. Bottom graph: $z=g(x,y) = 2+x^2$

So, $\text{Vol} = \iint_R (f(x,y) - g(x,y)) \, dA = \iint_R (6-y) \, dA - \iint_R (2+x^2) \, dA$. We compute each integral using Fubini; for this we view $R$ as an area between the curves $y_1(x) = 4-x^2$ and $y_2(x) = 0$. They meet at $x=2$ and $x=-2$.

So we integrate first with respect to $y$ (inside) and then with respect to $x$. 

\[\text{Vol} = \int_{-2}^{2} \left[ \int_{0}^{4-x^2} (6-y) \, dy \right] \, dx - \int_{-2}^{2} \left[ \int_{0}^{4-x^2} (2+x^2) \, dy \right] \, dx\]
Problem 5. We start by drawing the solid. Notice that the bounding surface can be nicely written in polar coordinates.

We compute the curve where both surfaces meet.

\[
\begin{align*}
    z &= x^2 + y^2 \\
    z &= 2 - x - y^2
\end{align*}
\]

As in Problem 5, we divide the solid in between the graphs of two functions defined on

\[ R = \{(x,y) : x^2 + y^2 \leq 1 \} \]

But we can compute the volume with polar coordinates:

\[
\text{Vol} = \iint_R (2 - r^2) \cdot r \, dr \, d\theta = 2\int_0^{2\pi} \int_0^1 r^2 \cdot r \, dr \, d\theta = \frac{\pi}{2}
\]