1. Let $D$ be a bounded object enclosed by the planes $x = y$, $z = y - 5$, $z = 0$ and $x = 0$, with density function $f(x, y, z) = |x + y + z| + 1$. Find:
   (a) the total mass of $D$; 
   (b) the $y$-coordinate of the centroid of $D$.

2. Evaluate $\iint_R y^2 \, dA$ where $R$ is the region bounded by the curves $x = y^2$, $x = y^2 - 4$, $x = 9 - y^2$ and $x = 16 - y^2$ with a suitable transformation suggested by the region. (Hint: Draw the picture.)

3. Compute the following line integrals
   (a) $\int_C xy \, ds$ where $C$ is the line segment from the point $(1, 2)$ to the point $(3, 5)$.
   (b) $\int_C xy \, ds$ where $C$ is the lower half of the circle of radius 1 about the origin starting at the point $(1, 0)$ and ending at the point $(-1, 0)$.

4. Let $\mathbf{F}$ be the force field $\mathbf{F}(x, y) = \left(\frac{1}{y}, 0\right)$. Compute the work required to move from the point $(1, 1)$ to the point $(-1, 1)$ along:
   (a) a straight line segment; 
   (b) a circle of radius $\sqrt{2}$. 
5. Consider the radial vector field \( \mathbf{F}(x, y) = (x, y) \). Find the flux of this vector field through:
   (a) the square with side length 4 centered at the origin ;  
   (b) a circle with radius \( R \) with center \((0,0)\).

6. Let \( \mathbf{F} \) be the vector field \( \mathbf{F}(x, y) = (-y, x) \).
   (a) Find the circulation of the vector field over a circle of radius \( r \) centered at the origin.
   (b) Let \( C \) be the loop bounding the polar rectangle
   \[
   R = \{(r, \theta) \mid a \leq r \leq b \text{ and } \alpha \leq \theta \leq \beta\}
   
   \text{oriented counter-clockwise.}
   
   i. Do you expect the circulation of } \mathbf{F} \text{ about } C \text{ to be positive, negative or zero?}
   
   ii. Compute the circulation of } \mathbf{F} \text{ about } C.
   
   iii. For \( p \geq 0 \) let } \mathbf{F}_p \text{ be the vector field
   \[
   \mathbf{F}_p(x, y) = \left( \frac{-y}{(x^2 + y^2)^p}, \frac{x}{(x^2 + y^2)^p} \right).
   
   Is there a value of } p \text{ for which the circulation of } \mathbf{F}_p \text{ is 0 for } C?\

7. Decide if the vector field \( \mathbf{F} \) is conservative on the given domain and if so, find the corresponding potential function.
   (a) \( \mathbf{F}(x, y) = (3x^2y^2, 2x^3y) \) on the domain \( \mathbb{R}^2 \).
   (b) \( \mathbf{F}(x, y) = \left( \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right) \) on the domain \( \{(x, y) \in \mathbb{R}^2 \mid x > 0\} \).
   (c) \( \mathbf{F}(x, y, z) = (y^2z^3, 2xyz^3 + 6yz, 3xy^2z^2 + 3y^2) \) on the domain \( \mathbb{R}^3 \).