MATH 2153 - Calculus III – Recitation 10

Prof. Cueto - The Ohio State University

April 7, 2016

1. Let D be a bounded object enclosed by the planes $x = y$, $z = y - 5$, $z = 0$ and $x = 0$, with density function $f(x, y, z) = |x + y + z| + 1$. Find:
   (a) the total mass of D ;
   (b) the y-coordinate of the centroid of D.

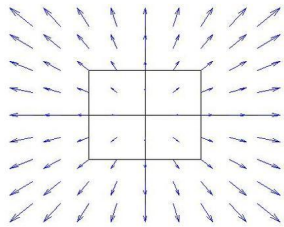
2. Evaluate $\iint_R y^2 \, dA$ where R is the region bounded by the curves $x = y^2$, $x = y^2 - 4$, $x = 9 - y^2$ and $x = 16 - y^2$ with a suitable transformation suggested by the region. (*Hint:* Draw the picture.)

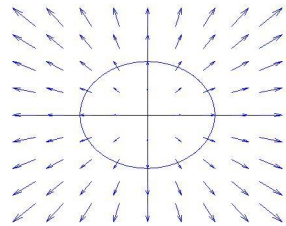
3. Compute the following line integrals
   (a) $\int_C xy \, ds$ where C is the line segment from the point $(1, 2)$ to the point $(3, 5)$.
   (b) $\int_C xy \, ds$ where C is the lower half of the circle of radius 1 about the origin starting at the point $(1, 0)$ and ending at the point $(-1, 0)$.

4. Let $\mathbf{F}$ be the force field $\mathbf{F}(x, y) = \langle \frac{1}{y}, 0 \rangle$. Compute the work required to move from the point $(1, 1)$ to the point $(-1, 1)$ along:
   (a) a straight line segment ;
   (b) a circle of radius $\sqrt{2}$.

5. Consider the radial vector field $\mathbf{F}(x, y) = \langle x, y \rangle$. Find the flux of this vector field through:

(a) the square with side length 4 centered at the origin ; (b) a circe with radius R with center $(0, 0)$.





6. Let $\mathbf{F}$ be the vector field $\mathbf{F}(x, y) = \langle -y, x \rangle$.

(a) Find the circulation of the vector field over a circle of radius r centered at the origin.

(b) Let C be the loop bounding the polar rectangle

$$R = \{(r, \theta) \,|\, a \leq r \leq b \text{ and } \alpha \leq \theta \leq \beta\}$$

oriented counter-clockwise.

i. Do you expect the circulation of $\mathbf{F}$ about C to be positive, negative or zero?

ii. Compute the circulation of $\mathbf{F}$ about C.

iii. For $p \geq 0$ let $\mathbf{F}_p$ be the vector field

$$\mathbf{F}_p(x, y) = \left\langle \frac{-y}{(x^2 + y^2)^p}, \frac{x}{(x^2 + y^2)^p} \right\rangle.$$

Is there a value of p for which the circulation of $\mathbf{F}_p$ is 0 for C?

7. Decide if the vector field $\mathbf{F}$ is conservative on the given domain and if so, find the corresponding potential function.

(a) $\mathbf{F}(x, y) = \langle 3x^2y^2, 2x^3y \rangle$ on the domain $\mathbb{R}^2$.

(b) $\mathbf{F}(x, y) = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$ on the domain $\{(x, y) \text{ in } \mathbb{R}^2 \,|\, x > 0\}$.

(c) $\mathbf{F}(x, y, z) = \langle y^2z^3, 2xyz^3 + 6yz, 3xy^2z^2 + 3y^2 \rangle$ on the domain $\mathbb{R}^3$.