MATH 2153 - Calculus III – Recitation 11

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1. Decide if the vector field $\mathbf{F}$ is conservative on the given domain and if so, find the corresponding potential function.

   (a) $\mathbf{F}(x, y) = \langle 3x^2y^2, 2x^3y\rangle$ on the domain $\mathbb{R}^2$.

   (b) $\mathbf{F}(x, y) = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle$ on the domain $\{(x, y) \text{ in } \mathbb{R}^2 \,|\, x > 0\}$.

   (c) $\mathbf{F}(x, y, z) = \langle y^2z^3, 2xyz^3 + 6yz, 3xy^2z^2 + 3y^2\rangle$ on the domain $\mathbb{R}^3$.

2. Fix $\mathbf{F}(x, y) = \langle f(x, y), g(x, y)\rangle$ a vector field defined on a region R in $\mathbb{R}^2$, where f and g have continuous first partials. We say a function $\psi\colon R \to \mathbb{R}$ is a *stream function* for $\mathbf{F}$ if $\dfrac{\partial \psi}{\partial y} = f$ and $\dfrac{\partial \psi}{\partial x} = -g$.

   (a) Prove that if ψ is a stream function for $\mathbf{F}$ then divergence($\mathbf{F}$)= 0.

(b) if R is connected and simply connected, then prove that if divergence($\mathbf{F}$)$= 0$, then $\mathbf{F}$ admits a stream function.

(c) Prove that the vector field $\mathbf{F}$ is tangent to the level curves of the stream functions, which we call *flow curves.*

3. Let $\mathbf{F} = \langle -y, 0 \rangle$ be a vector field defined on $\mathbb{R}^2$ and let C be the closed loop bounding the triangle with vertices $(-1, 1)$, $(3, 2)$, and $(0, 4)$ oriented counterclockwise.

   (a) Evaluate the circulation of the vector field $\mathbf{F}$ along C.

   (b) Use Green's theorem to give a double integral which computes this circulation.

   (c) Are line integrals in the vector field $\mathbf{F}$ path independant in the domain $\mathbb{R}^2$?

4. Find the circulation and flux of the vector field $\mathbf{F}(x, y, z) = \langle z, x, -y \rangle$ on the tilted ellipse C parameterized by $\mathbf{r}(t) = \langle \cos t, \sin t, \cos t \rangle$ for $0 \le t \le 2\pi$. (*Hint:* Find the plane containing C to define the outernormal.)