Lecture 10: §2.1 Vectors in the Plane & §2.2 Vectors in Space

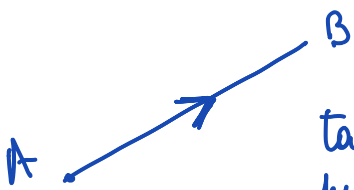
Recall: Vectors in $\mathbb{R}^n$ = $n \times 1$ matrices

In $\mathbb{R}^2$: $\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix}$, etc. In $\mathbb{R}^3$: $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, etc.

TODAY: How can we represent a vector geometrically?

Why? Use vectors to model Physics (e.g. force, displacement, velocity, etc...)

Q: What is a vector in $\mathbb{R}^2$ or $\mathbb{R}^3$?



$\vec{v} = \overrightarrow{AB}$

tail = A = initial pt

head = B = terminal pt.

Observations:

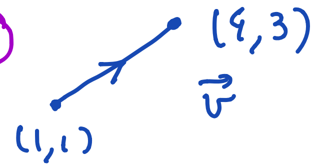
① We do not distinguish between parallel vectors of the same length

② $\vec{0} = \cdot$ is the only vector of magnitude 0. It has <u>no</u> direction.

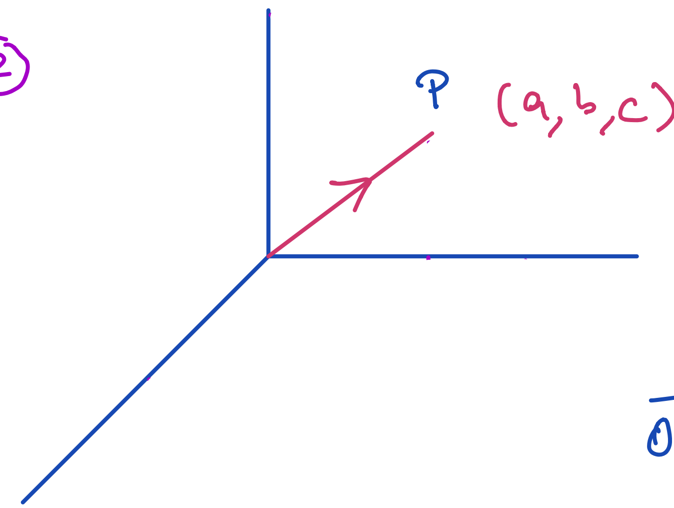
<u>Q</u>: Can we have a consistent choice to represent all these vectors?

<u>A</u>:

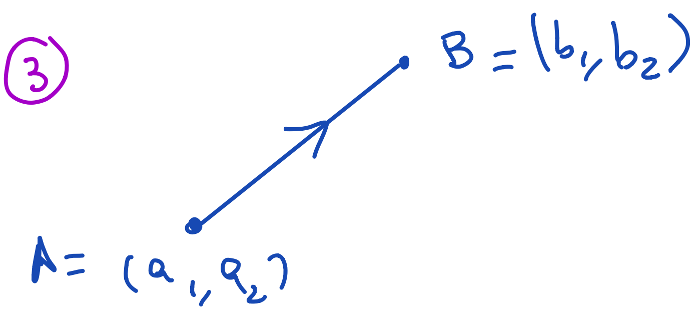
Examples

① 

⇝ position vector

② 

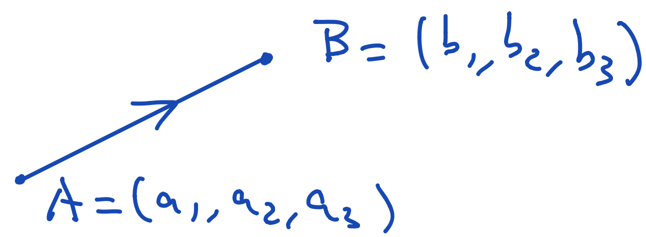
$\vec{OP} = \begin{bmatrix} \\ \end{bmatrix}$

③ 

⇝ x-component of $\vec{v}$ =

y-component of $\vec{v}$ =

$\vec{OP} = \begin{bmatrix} \\ \\ \end{bmatrix}$ position vector for $\vec{AB}$

④ 

⇝ 3 components:

$\begin{cases} \text{x-comp} = \\ \text{y-comp} = \\ \text{z-comp} = \end{cases}$

⇝ $\vec{OP} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$ position vector

Q: Why use position vectors?

A

Example: Given $v = \overrightarrow{AB}$ with position vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, find the coordinates of B given $A = (3, 4)$ & draw $\vec{v}$

Recall: Magnitude of $\vec{v}$ $= \begin{cases} \sqrt{v_1^2 + v_2^2} & \text{if } \vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \text{ in } \mathbb{R}^2 \\ \sqrt{v_1^2 + v_2^2 + v_3^2} & \text{if } \vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \text{ in } \mathbb{R}^3 \end{cases}$

Operations on vectors:
1. Addition
2. Scalar Multiplication

Q: How to do them geometrically?

. See these operations for $\mathbb{R}^2$, but the same idea works for $\mathbb{R}^3$.

① $\boxed{\text{ADDITION}}$

$\vec{u} + \vec{v} = ?$ Two ways:

- <u>Triangle Law</u>: Put Tail of $\vec{v}$ at the head of $\vec{u}$ & draw Δ.

- <u>Parallelogram Law</u>: Put 2 tails at the same point P, draw parallelogram & the diagonal from P to opposite corner.

② $\boxed{\text{SCALAR MULT.}}$

$\left.\begin{array}{ll} \vec{v} & \text{vector} \\ \alpha & \text{scalar} \end{array}\right\} \rightsquigarrow \alpha\vec{v}$ is a vector with

$\left\{\begin{array}{l} \cdot \ \text{magnitude}(\alpha\vec{v}) = \\ \cdot \ \text{direction}(\alpha\vec{v}) = \end{array}\right.$

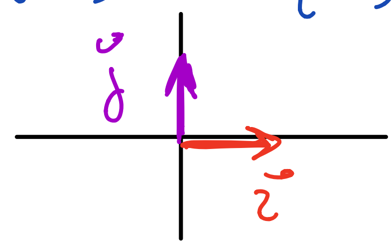
③ Substraction / Difference

Triangle Law:

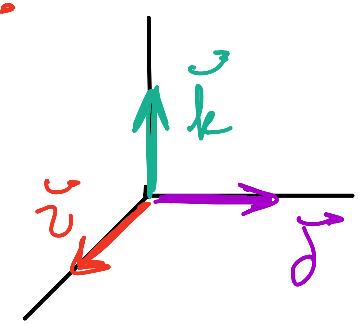
Parallelogram Law: Take the 2^{nd} diagonal!

Recall: Basic unit vectors (in $\mathbb{R}^n$) $e_1 = \begin{bmatrix}1\\0\\\vdots\\0\end{bmatrix}, e_2 = \begin{bmatrix}0\\1\\0\\\vdots\\0\end{bmatrix}, \dots, e_n = \begin{bmatrix}0\\\vdots\\0\\1\end{bmatrix}$

For $\mathbb{R}^2$: $\vec{i} = \begin{bmatrix}1\\0\end{bmatrix}, \vec{j} = \begin{bmatrix}0\\1\end{bmatrix}$



For $\mathbb{R}^3$: $\vec{i} = \begin{bmatrix}1\\0\\0\end{bmatrix}$ $\vec{j} = \begin{bmatrix}0\\1\\0\end{bmatrix}$ $\vec{k} = \begin{bmatrix}0\\0\\1\end{bmatrix}$



Proposition Any $\vec{v}$ in $\mathbb{R}^n$ is a linear combination of basic unit vectors

Example Solve $\quad a\begin{bmatrix}1\\2\\3\end{bmatrix} + b\begin{bmatrix}4\\0\\-1\end{bmatrix} = \begin{bmatrix}1\\-2/3\\-4/3\end{bmatrix}$ for a, b in $\mathbb{R}$

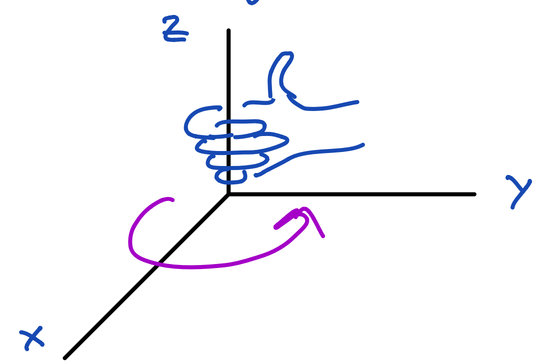
<u>Example 1</u>: Find all vectors of length 1 parallel to $\vec{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

<u>Soln</u>:

<u>Example 2</u>: Same question, but now require length 8.

Rectangular Coordinates in $\mathbb{R}^3$

The three coordinate axes are directed according to the right hand rule



3 coordinate planes :

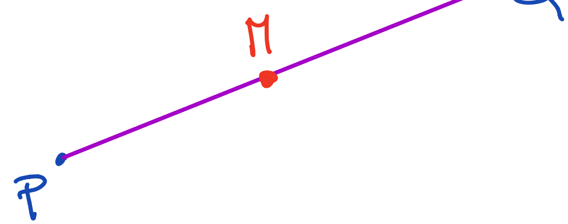
Plane	Which axes	Equation
XY		
XZ		
YZ		

Planes parallel to coordinate planes = (1)

(2)

(3)

Q: Midpoint between P & Q ?



$P = (p_1, p_2, p_3)$

$Q = (q_1, q_2, q_3)$