Sec 1.3

4) $\left[\begin{array}{cccc|c} 1 & 2 & 3 & 2 & 1 \\ 1 & 2 & 3 & 5 & 2 \\ 2 & 4 & 6 & 1 & 1 \\ -1 & -2 & -3 & 7 & 2 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} 1 & 2 & 3 & 0 & 1/3 \\ 0 & 0 & 0 & 1 & 1/3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$r=2$ $n=4$ so 2 independent variables
x_2, x_3

6) $\left[\begin{array}{cccc|c} a_{11} & a_{12} & a_{13} & a_{14} & b_1 \\ a_{21} & a_{22} & a_{23} & a_{24} & b_2 \\ a_{31} & a_{32} & a_{33} & a_{34} & b_3 \end{array}\right]$

$r=2$ one row is parallel to another, x_3, x_4 independent
$r=3$ x_4 is a independent variable

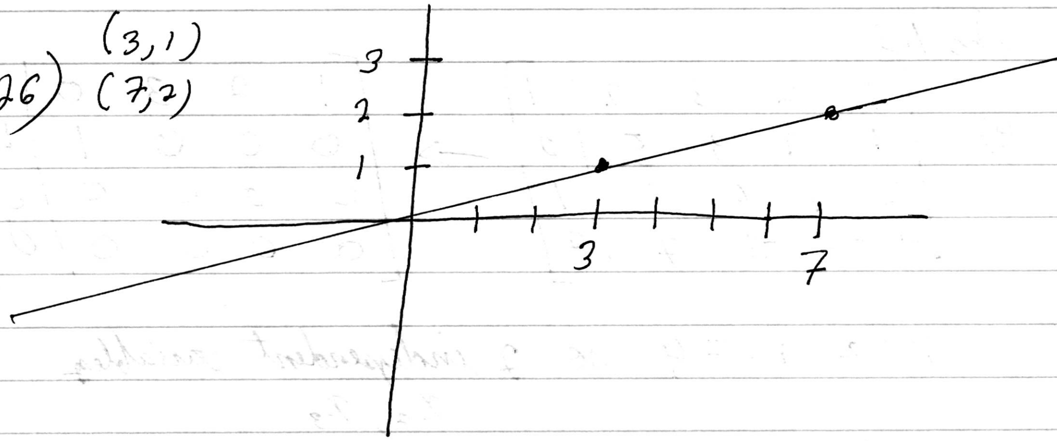
10) A system of 4 equations in 3 unknowns

could be any of the possibilities

14) A system 3 equations in 4 unknowns that has $x_1=-1$, $x_2=0$, $x_3=2$, $x_4=-3$ as a solution

$m=3$ $n=4$ so $m<n$ by the set of solutions and corollary in the section, the has infinite many solutions

26) (3,1) (7,2)



$$dx + ey + f = 0$$

$$\begin{cases} 3d + y + f = 0 \\ 7d + 2y + f = 0 \end{cases}$$

$$\left[\begin{array}{ccc|c} 3 & 1 & 1 & 0 \\ 7 & 2 & 1 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 4 & 0 \end{array}\right]$$

$d = f \qquad e = -4f$

$$fx - 4fy + f = 0 \qquad \boxed{\frac{x+1}{4} = y}$$

28) $(-4,0), (-2,-2), (0,3), (1,1), (4,0)$

$$ax^2 + bxy + cy^2 + dx + ey + f = 0$$

$$16a + 0 + 0 + (-4)x + 0 + f = 0$$
$$4a + 4b + 4c + (-2)d + (-2)e + f = 0$$
$$0 + 0 + 9c + 0 + 3e + f = 0$$
$$a + b + c + d + e + f = 0$$
$$16a + 0 + 0 + 4d + 0 + f = 0$$

$$\left[\begin{array}{cccccc|c} 16 & 0 & 0 & -4 & 0 & 1 & 0 \\ 4 & 4 & 4 & -2 & -2 & 1 & 0 \\ 0 & 0 & 9 & 0 & 3 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 16 & 0 & 0 & 4 & 0 & 1 & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 0 & 1/16 & 0 \\ 0 & 1 & 0 & 0 & 0 & 71/144 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1/2 & 0 \end{array}\right]$$

$a = -\frac{1}{16}f$
$b = -71/144 f$
$c = 1/18 f$
$d = 0$
$e = -1/2 f$

$$\boxed{-\frac{1}{16}x^2 - \frac{71}{144}xy + \frac{1}{18}y^2 - \frac{1}{2}y + 1 = 0}$$

Sec 1.5

8) (a) $t + s = \begin{bmatrix} 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

(b) $r + 3u = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 3\begin{bmatrix} -4 \\ 6 \end{bmatrix} = \begin{bmatrix} -11 \\ 18 \end{bmatrix}$

(c) $2u + 3t = 2\begin{bmatrix} -4 \\ 6 \end{bmatrix} + 3\begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} -5 \\ 24 \end{bmatrix}$

14) $a_1 r + a_2 s = u$

$\left[\begin{array}{cc|c} 1 & 2 & -4 \\ 0 & -3 & 6 \end{array}\right]$

$\boxed{a_2 = -2}$

$a_1 + 2a_2 = -4$

$\boxed{a_1 = -4 - 2(-2) = 0}$

$-2s = u$

22) $w_1 = Cs \qquad w_2 = Aw_1$

$\begin{bmatrix} -2 & 3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} -13 \\ -1 \end{bmatrix} = w_1$

$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} -13 \\ -1 \end{bmatrix} = \begin{bmatrix} -27 \\ -16 \end{bmatrix} = w_2$

$$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 7 \\ 1 & 6 \end{bmatrix} = Q$$

$$\begin{bmatrix} -3 & 7 \\ 1 & 6 \end{bmatrix}\begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} -27 \\ -16 \end{bmatrix} = W_2$$

34) $$uv = \begin{bmatrix} 1 \\ 3 \end{bmatrix}\begin{bmatrix} 2 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 6 & 12 \end{bmatrix}$$

$$vu = \begin{bmatrix} 2 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} = 14$$

42) $$\begin{bmatrix} 1 & 0 & -1 & -2 & 0 \\ 0 & 1 & 2 & 3 & 0 \end{bmatrix}$$

let $x_4 = \alpha \quad x_3 = \beta$

$$x_2 = -2x_3 - 3x_4 = -2\beta - 3\alpha$$
$$x_1 = x_3 + 2x_4 = \beta + 2\alpha$$

$$X = \begin{bmatrix} \beta + 2\alpha \\ -2\beta - 3\alpha \\ \beta \\ \alpha \end{bmatrix} = \alpha\begin{bmatrix} 2 \\ -3 \\ 0 \\ 1 \end{bmatrix} + \beta\begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}$$

α, β any scalar

48) $\left[\begin{array}{cccccc|c} 1 & 0 & -1 & 0 & -1 & -2 & 0 \\ 0 & 1 & 2 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$

let $x_6 = \alpha$, $x_5 = \beta$, $x_3 = \gamma$

$x_4 = -\alpha - \beta$

$x_2 = -2\alpha - \beta - 2\gamma$

$x_1 = 2\alpha + \beta + \gamma$

$$X = \begin{bmatrix} 2\alpha+\beta+\gamma \\ -2\alpha-\beta-2\gamma \\ \gamma \\ -\alpha-\beta \\ \beta \\ \alpha \end{bmatrix} = \alpha \begin{bmatrix} 2 \\ -2 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ -1 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} + \gamma \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

α, β, γ any scalar

54) $A(2\times 3)$ $B(3\times 4)$ $C(4\times 4)$ $D(4\times 2)$

$(AB)(CD) = (2\times 4)(4\times 2) = (2\times 2)$

$A[B(CD)] = (2\times 3)[(3\times 4)(4\times 2)]$

$= (2\times 3)(3\times 2) = (2\times 2)$

68) $\left[\begin{array}{ccccc|c} 1 & 3 & -3 & 2 & -3 & -4 \\ 3 & 9 & -10 & 10 & -14 & 2 \\ 2 & 6 & -10 & 21 & -25 & 53 \end{array}\right]$

$\Rightarrow \left[\begin{array}{ccccc|c} 1 & 3 & 0 & 0 & 22 & 4 \\ 0 & 0 & 1 & 0 & 9 & 6 \\ 0 & 0 & 0 & 1 & 1 & 5 \end{array}\right]$

let $x_5 = \alpha$, $x_2 = \beta$

$x_4 = 5 - \alpha$
$x_3 = 6 - 9\alpha$
$x_1 = 4 - 22\alpha - 3\beta$

$$X = \begin{bmatrix} 4 - 22\alpha - 3\beta \\ \beta \\ 6 - 9\alpha \\ 5 - \alpha \\ \alpha \end{bmatrix} = \alpha \begin{bmatrix} -22 \\ 0 \\ -9 \\ -1 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 4 \\ 0 \\ 6 \\ 5 \\ 0 \end{bmatrix}$$