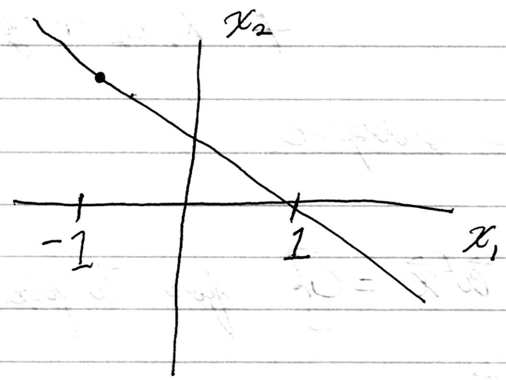
3.1.12

$$W = \left\{ \vec{x} : \vec{x} = \begin{bmatrix} a \\ b \end{bmatrix}, a+b=1 \right\}$$

$$a = 1-b$$

$$\vec{x} = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1-b \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + b\begin{bmatrix} -1 \\ 1 \end{bmatrix}$$



3.2.6

$$W = \{ \vec{x} : |x_1| + |x_2| = 0 \}$$

(S1) let $\vec{x} = \vec{0}$, then $|x_1| + |x_2| = |0| + |0| = 0$

(S2) $\vec{x} + \vec{y} \Rightarrow \quad |x_1 + y_1| + |x_2 + y_2| \leq |x_1| + |y_1| + |x_2| + |y_2|$

So is not a subspace

3.2.8 $W = \{\vec{x} : x_1 x_2 = 0\}$

(S1) let $\vec{x} = \vec{0}$

$$x_1 x_2 = (0)(0) = 0$$

(S2) $\vec{x} + \vec{y} \Rightarrow (x_1 + y_1)(x_2 + y_2) = x_1 x_2 + x_1 y_2 + y_1 x_2 + y_1 y_2$

$$\neq x_1 x_2 + y_1 y_2$$

so is not a subspace

3.2.18 $W = \{\vec{x} : \vec{a}^T \vec{x} = 0\}$ for $\vec{a}$ fixed in $\mathbb{R}^3$

(S1) let $\vec{x} = \vec{0}$

$$\vec{a}^T \vec{x} = \vec{a}^T 0 = 0$$

(S2) $\vec{a}^T (\vec{x} + \vec{y}) = \vec{a}^T \vec{x} + \vec{a}^T y$

$$= 0 + 0 = 0$$

(S3) $\vec{a}^T (\alpha \vec{x})$ for some constant α

$$\Rightarrow \alpha (\vec{a}^T \vec{x}) = \alpha (0) = 0$$

$\therefore$ this is a subspace of $\mathbb{R}^3$

3.2.30 U and V subsets of $\mathbb{R}^n$

$U+V = \{\vec{x} : \vec{x} = \vec{u} + \vec{v}\,;\ \vec{u} \in U \text{ and } \vec{v} \in V\}$

Let U and V be subspaces of $\mathbb{R}^n$

(S1) then there is $\vec{u} = \vec{0} \in U$ and
$\vec{v} = \vec{0} \in V$
so $\vec{u} + \vec{v} = \vec{0} + \vec{0} = \vec{0} \in U+V$

(S2) Consider $\vec{x} + \vec{y} = (\vec{u}_x + \vec{v}_x) + (\vec{u}_y + \vec{v}_y)$

$$= (\vec{u}_x + \vec{u}_y) + (\vec{v}_x + \vec{v}_y)$$

Since U and V are subspaces
$\vec{u}_x + \vec{u}_y \in U$ and $\vec{v}_x + \vec{v}_y \in V$

$\therefore\ \vec{x} + \vec{y} \in U+V$

(S3) Consider $\alpha\vec{x}$ for some constant α

then $\alpha\vec{x} = \alpha(u + v) = \alpha u + \alpha v$

Since U and V are subspaces

$\alpha u \in U$ and $\alpha v \in V$

$\therefore\ \alpha\vec{x} \in U+V$

$\therefore\ U+V$ is a subspace