3.4-16

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 2 & 2 & 1 \\ 2 & 3 & 0 \end{bmatrix}$$

(a) $$\begin{bmatrix} 2 & 1 & 2 \\ 2 & 2 & 1 \\ 2 & 3 & 0 \end{bmatrix} \begin{matrix} \\ -1 \\ -1 \end{matrix} \to \begin{bmatrix} 2 & 1 & 2 \\ 0 & -1 & 1 \\ 0 & -2 & 2 \end{bmatrix} \begin{matrix} \\ -2 \\ \end{matrix}$$

$$\to \begin{bmatrix} 2 & 1 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} = B$$

(b) $$\begin{bmatrix} 2 & 1 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$2x_1 + x_2 + 2x_3 = 0$$

$$-x_2 + x_3 = 0 \qquad x_2 = x_3$$

$$2x_1 = -x_2 - 2x_3 = -3x_3 \qquad x_1 = -\frac{3}{2}x_3$$

$$\vec{X} = x_3 \begin{bmatrix} -3/2 \\ 1 \\ 1 \end{bmatrix}$$

(c) $$\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

New $$\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = \frac{3}{2} \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} - 1 \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Then $c_1 \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\begin{array}{c} \\ -1 \\ -1 \end{array} \left[\begin{array}{cc|c} 2 & 1 & 0 \\ 2 & 2 & 0 \\ 2 & 3 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 2 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & -2 & 0 \end{array}\right] \quad \begin{array}{l} c_1 = 0 \\ \\ c_2 = 0 \end{array}$$

$\therefore$ L.I.

so $\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is a basis

(d) Take $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

$$c_1 \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{c} \\ -2 \\ -1 \end{array} \left[\begin{array}{cc|c} 2 & 0 & 0 \\ 1 & -1 & 0 \\ 2 & 1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & -1 & 0 \end{array}\right] \quad \begin{array}{l} c_1 = c_2 = 0 \\ \\ \text{so L.I.} \end{array}$$

and since $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$ can

be writen in terms of these two vectors

the they form a basis

3.4-36

$$\vec{V}_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \qquad \vec{V}_2 = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$$

let $w = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$$c_1 \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$2 \quad -2 \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 2 & -1 & 2 \\ -1 & -2 & 3 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & -5 & 0 \\ 0 & 2 & 5 \end{array}\right] \begin{array}{c} \\ 2 \\ 3 \end{array}$$

$$\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & -5 & 0 \\ 0 & 0 & 25 \end{array}\right]$$ so is inconcistent

so w is not a linear comb. of $\vec{V}_1$ and $\vec{V}_2$

3.5-20

$$\begin{aligned} x_1 - x_2 \quad\quad\quad &= 0 \\ x_2 - 2x_3 \quad &= 0 \\ x_3 - x_4 &= 0 \end{aligned}$$

$x_3 = x_4$

$x_2 = 2x_3 = 2x_4$

$x_1 = x_2 = 2x_4$

$$\vec{X} = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 1 \end{bmatrix} x_4$$

$\dim(W) = 1$

3.5-22

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 2 & -5 & 1 \end{bmatrix}$$

$$2 \left[\begin{array}{ccc|c} -1 & 2 & 0 & 0 \\ 2 & -5 & 1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} -1 & 2 & 0 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right]$$

$$\begin{aligned} -x_1 + 2x_2 \quad &= 0 \\ -x_2 + x_3 &= 0 \end{aligned} \qquad \begin{aligned} x_1 &= 2x_2 = 2x_3 \\ x_2 &= x_3 \end{aligned}$$

$$\vec{X} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} x_3$$

nullity $= 1$

$n = 3$

rank $= 3 - 1 = 2$

3.5 - 26

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 2 & 4 & 2 & 4 \\ 2 & 1 & 5 & -2 \end{bmatrix}$$

$$-2\left[\begin{array}{cccc|c} 1 & 1 & 2 & 0 & b_1 \\ 2 & 4 & 2 & 4 & b_2 \\ 2 & 1 & 5 & -2 & b_3 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 2 & 0 & b_1 \\ 0 & 2 & -2 & 4 & b_2 - 2b_1 \\ 0 & -1 & 1 & -2 & b_3 - b_1 \end{array}\right] 2$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 0 & b_1 \\ 0 & 2 & -2 & 4 & b_2 - 2b_1 \\ 0 & 0 & 0 & 0 & 2b_3 - 2b_1 + b_2 - 2b_1 \end{array}\right]$$

$$-4b_1 + b_2 + 2b_3 = 0 \qquad b_3 = \frac{4b_1 - b_2}{2}$$

$$\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ 2b_1 - \frac{1}{2}b_2 \end{bmatrix} = b_1 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + b_2 \begin{bmatrix} 0 \\ 1 \\ -1/2 \end{bmatrix}$$

rank = dim $(R(A)) = 2$

$n = 4$

nullity $= 4 - 2 = 2$