5.8 - 2 $T: P_3 \longrightarrow P_4$ s.t. $T(P) = (x+2)P(x)$

$2T(P) = 2(x+2)P(x)$

$2T(x) = 2(x+2)x = 2x^2 + 4x$

5.8 - 10 $T: V \longrightarrow V$ s.t. $T(A) = Q^{-1}AQ$

where Q is nonsingular

Since Q is nonsingular $Q \neq 0$

then $T(A) = 0 = Q^{-1}AQ$

$0 = A$

so $N(T) = \{0\} \Rightarrow$ nullity $= 0$

so $\dim R(T) = \dim V$ so T is onto

$\therefore$ T is invertible

$$\begin{aligned} T^{-1}(Q^{-1}AQ) &= Q(Q^{-1}AQ)Q^{-1} \\ &= (QQ^{-1})A(QQ^{-1}) \\ &= A \end{aligned}$$

$T^{-1}(B) = QBQ^{-1}$ $B \in R(T)$

5.9-2 $\quad T: P_3 \to P_4$ s.t. $T(p) = (x+2)p(x)$

$$B = \{1, x, x^2, x^3\} \qquad C = \{1, x, x^2, x^3, x^4\}$$

$$T(1) = (x+2)(1) = x+2$$
$$T(x) = (x+2)(x) = x^2 + 2x$$
$$T(x^2) = (x+2)(x^2) = x^3 + 2x^2$$
$$T(x^3) = (x+2)(x^3) = x^4 + 2x^3$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$