Quiz 1

NOTE: Answers without proper justfication will receive NO credit

Problem 1. *(2 points)* Find three numbers whose sum is 34, when the sum of the first and second is 7, and the sum of the second and third is 22.

$$X_1 + X_2 + X_3 = 34$$
$$X_1 + X_2 = 7$$
$$X_2 + X_3 = 22$$

$$\xrightarrow[E_3 - E_1]{E_1 - E_2}$$

$$\begin{cases} X_3 = 34 - 7 = 27 \\ X_1 + X_2 = 7 \\ X_1 = 34 - 22 = 12 \end{cases}$$

$$\Rightarrow X_2 = 7 - X_1 = 7 - 12 = -5$$

$$\boxed{\underline{Soln:} \quad X_1 = 12, \quad X_2 = -5, \quad X_3 = 27.}$$

Problem 2. *(3 points)* Find the unique reduced echelon form of the augmented matrix associated to the system:
$$\begin{cases} x_1 & + x_3 + x_4 - 2x_5 = 1 \\ 2x_1 + x_2 + 3x_3 - x_4 + x_5 = 0 \\ 3x_1 - x_2 + 4x_3 + x_4 + x_5 = 1 \end{cases}.$$

$$B = \begin{bmatrix} 1 & 0 & 1 & 1 & -2 & | & 1 \\ 2 & 1 & 3 & -1 & 1 & | & 0 \\ 3 & -1 & 4 & 1 & 1 & | & 0 \end{bmatrix} \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 0 & 1 & 1 & -2 & | & 1 \\ 0 & 1 & 1 & -3 & 5 & | & -2 \\ 0 & -1 & 1 & -2 & 7 & | & -2 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 + R_2} \begin{bmatrix} 1 & 0 & 1 & 1 & -2 & | & 1 \\ 0 & 1 & 1 & -3 & 5 & | & -2 \\ 0 & 0 & 2 & -5 & 12 & | & -4 \end{bmatrix} \xrightarrow{R_3 \to \frac{R_3}{2}} \begin{bmatrix} 1 & 0 & 1 & 1 & -2 & | & 1 \\ 0 & 1 & 1 & -3 & 5 & | & -2 \\ 0 & 0 & 1 & -5/2 & 6 & | & -2 \end{bmatrix}$$

$$\xrightarrow[R_2 \to R_2 - R_3]{R_1 \to R_1 - R_3} \begin{bmatrix} 1 & 0 & 0 & -7/2 & -8 & | & 3 \\ 0 & 1 & 0 & -1/2 & -1 & | & 0 \\ 0 & 0 & 1 & -5/2 & 6 & | & -2 \end{bmatrix} \quad \text{REF frm!}$$