SOLUTIONS

Math 2568 (§75) – Feb. 22, 2017 Full Name: ______________________

Quiz 5

Answers without proper justification will receive NO credit.

Problem 1. *(2 points)* Find a basis for the row space of the matrix $\begin{bmatrix} 2 & 1 & 2 \\ 2 & 2 & 1 \\ 2 & 3 & 0 \end{bmatrix}$.

METHOD 1 We find the (R)EF of the matrix:

$$\begin{bmatrix} 2 & 1 & 2 \\ 2 & 2 & 1 \\ 2 & 3 & 0 \end{bmatrix} \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - R_1} \begin{bmatrix} 2 & 1 & 2 \\ 0 & 1 & -1 \\ 0 & 2 & -2 \end{bmatrix} \xrightarrow[R_3 \to R_3 - 2R_2]{} \begin{bmatrix} 2 & 1 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow[R_1 \to \frac{R_1}{2}]{} \begin{bmatrix} 1 & \frac{1}{2} & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{Basis} = \left\{ \begin{bmatrix} 1 \\ \frac{1}{2} \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$$

METHOD 2 Find dependencies

$$x_1 \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 2 & 2 & 2 \\ 1 & 2 & 3 \\ 2 & 1 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \\ 2 & 1 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -4 \\ 0 & -1 & -2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & -1 \\ 0 & -1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

Basis $\Leftrightarrow$ dependent variables so $B' = \left\{ \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} \right\}$

Problem 2. *(3 points)* Given the matrix $A = \begin{bmatrix} -1 & 2 & 0 \\ 2 & -5 & 1 \end{bmatrix}$:

(i) Find a basis for the null space $\mathcal{N}(A)$ and compute the nullity ($= \dim \mathcal{N}(A)$) of A.

$\mathcal{N}(A) = \{ x \mid Ax = 0 \}$

$$\begin{bmatrix} -1 & 2 & 0 \\ 2 & -5 & 1 \end{bmatrix} \xrightarrow[R_1 \to -R_1]{R_2 \to R_2 + 2R_1} \begin{bmatrix} 1 & -2 & 0 \\ 0 & -1 & 1 \end{bmatrix} \xrightarrow[R_2 \to \frac{R_2}{-1}]{R_1 \to R_1 - 2R_2} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \end{bmatrix}$$

$x_1 = 2x_3$
$x_2 = x_3$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$$

so Basis $\mathcal{N}(A) = \left\{ \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \right\}$

& $\boxed{\text{nullity} = 1}$

(ii) Compute the rank ($= \dim \mathcal{R}(A)$) of A.

By a Thm seen in class $\text{rk}(A) = \dim \mathcal{R}(A) = \dim \mathcal{R}(A^T) = \text{rk}(A^T)$. From (i), we know that A has 2 l.i. rows, so $\text{rk}(A^T) = 2$.

We conclude $\boxed{\text{rank}(A) = 2.}$

Alternatively, we find a basis for $\mathcal{R}(A)$:

$$\begin{bmatrix} -1 & 2 \\ 2 & -5 \\ 0 & 1 \end{bmatrix} \xrightarrow[R_2 \to R_2 + 2R_1]{} \begin{bmatrix} -1 & 2 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \xrightarrow[R_3 \to R_3 - R_2]{} \begin{bmatrix} -1 & 2 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \xrightarrow[R_1 \to -R_1]{} \begin{bmatrix} 1 & -2 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$ } basis

so a basis for $\mathcal{R}(A) = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$

& so $\text{rank}(A) = 2$.