Math 2568 (§75) – March 1, 2017   Full Name: ____________________

Quiz 6

Answers without proper justification will receive NO credit.

Problem 1. *(2 points)* Let $\mathbb{W}$ be the subset of $\mathcal{P}_3$={polynomials of degree ≤ 3} defined by

$$\mathbb{W} = \{p(x) \text{ in } \mathcal{P}_3 \colon p(1) = p(-1) \text{ and } p(2) = p(-2)\}.$$

Find a spanning set for $\mathbb{W}$ and conclude that $\mathbb{W}$ is a subspace of $\mathcal{P}_3$.

- Use operations $+$ & $\cdot$ from functions $\mathbb{R} \to \mathbb{R}$.
- $P = ax^3 + bx^2 + cx + d$

$$\left.\begin{array}{l} P(1) = a+b+c+d = P(-1) = -a+b-c+d \to 2(a+c)=0 \\ P(2) = 8a+b4+2c+d = P(-2) = -8a+4b-2c+d \to 2(8a+c)=0 \end{array}\right\} \text{conclude } \boxed{a=c=0}$$

so $P = bx^2 + d$ for any b, d in $\mathbb{R}$ $\leadsto$ clearly a subspace of $\mathcal{P}_3$

$$\boxed{\mathbb{W} = Sp(1, x^2)}$$

Problem 2. *(3 points)* Let $\mathbb{V}$ be the vector space of all (2×2) matrices. Find a basis for the subspace Sp(A_1, A_2, A_3, A_4) where

$$A_1 = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} -1 & -1 \\ 1 & -3 \end{bmatrix}, \quad \text{and} \quad A_4 = \begin{bmatrix} -2 & 2 \\ 2 & 0 \end{bmatrix}.$$

Use coordinates with respect to $B = \{E_{11}, E_{12}, E_{21}, E_{22}\}$

$$\begin{array}{l} [A_1]_B = \\ [A_2]_B = \\ [A_3]_B = \\ [A_4]_B = \end{array} \begin{bmatrix} 1 & 2 & -1 & 3 \\ -2 & 1 & 2 & -1 \\ -1 & -1 & 1 & -3 \\ -2 & 2 & 2 & 0 \end{bmatrix} \xrightarrow[\substack{R_2 \to R_2 + 2R_1 \\ R_3 \to R_3 + R_1 \\ R_4 \to R_4 + 2R_1}]{} \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 5 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 6 & 0 & 6 \end{bmatrix} \xrightarrow[\substack{R_2 \to R_2/5 \\ R_3 \to R_3 - R_2 \\ R_4 \to R_4 - 6R_2 \\ R_3 \to R_3/1}]{} \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & +1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

E.F.

coordinates of a basis:

$$[1 \ 2 \ -1 \ 3] = E_{11} + 2E_{12} - E_{21} + 3E_{22} = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$$

$$[0 \ 1 \ 0 \ 1] = E_{12} + E_{22} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$[0 \ 0 \ 0 \ +1] = +E_{12} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

We get a basis $B = \left\{ \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$.