Quiz 7

NOTE: Answers without proper justfication will receive NO credit

Give the set $M_{2\times 2}$ of 2×2 matrices, define a linear transformation $T: M_{2\times 2} \to \mathcal{P}_2$ by

$$T(E_{11}) = 1 - x, \quad T(E_{12}) = 1 + x + x^2, \quad T(E_{21}) = 2x - x^2, \quad T(E_{22}) = 2 + x - 2x^2.$$

Problem 1. *(2 points)* Compute $T\left(\begin{bmatrix} -2 & 2 \\ 3 & 4 \end{bmatrix}\right)$.

$$\begin{bmatrix} -2 & 2 \\ 3 & 4 \end{bmatrix} = -2E_{11} + 2E_{12} + 3E_{21} + 4E_{22}$$

$$So\ T\begin{bmatrix} -2 & 2 \\ 3 & 4 \end{bmatrix} = -2(1-x) + 2(1+x+x^2) + 3(2x-x^2) + 4(2+x-2x^2)$$

$$= (-2+2+8) + (2+2+6+4)x + (2-3-8)x^2$$

$$= 8 + 14x - 9x^2$$

Problem 2. *(3 points)* Give a formula for $T(A)$ in terms of the four entries of $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a(1-x) + b(1+x+x^2) + c(2x-x^2) + d(2+x-2x^2)$$

$$= (a+b+2d) + (-a+b+2c+d)x + (b-c-2d)x^2$$