Math 2568 (§75) – March. 31, 2017 Full Name: ______________________

Quiz 8

NOTE: Answers without proper justfication will receive NO credit

Consider two linear transformations between spaces of polynomials of degrees ≤3 ($\mathcal{P}_3$) and ≤4 ($\mathcal{P}_4$):

$H\colon \mathcal{P}_4 \to \mathcal{P}_3$ where $H(p(x)) = p'(x) + p(0)$ and $T\colon \mathcal{P}_3 \to \mathcal{P}_4$ where $T(p(x)) = (x+2)p(x)$.

Consider the natural bases $B = \{1, x, x^2, x^3\}$ for $\mathcal{P}_3$ and $C = \{1, x, x^2, x^3, x^4\}$ for $\mathcal{P}_4$.

Problem 1. *(3 points)* Find the matrix for T with respect to the bases B and C.

$T(1) = x+2$

$T(x) = (x+2)x = x^2+2x$

$T(x^2) = (x+2)x^2 = x^3+2x^2$

$T(x^3) = (x+2)x^3 = x^4+2x^3$

$$[T]_{BC} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 2. *(2 points)* Given the formula for the composition $H \circ T\colon \mathcal{P}_3 \to \mathcal{P}_3$.

$$H(T(p)) = H\big((x+2)P(x)\big) = \big((x+2)P(x)\big)' + \big((x+2)P(x)\big)(0)$$

$$= P(x) + xP'(x) + 2P'(x) + 2P(0).$$

In coordinates: $P = a + bx + cx^2 + dx^3$

$$H\circ T(p) = a + bx + cx^2 + dx^3 + (2+x)(b + 2cx + 3dx^2) + 2a$$

$$= 3a + bx + cx^2 + dx^3 + 2b + 4cx + 6dx^2 + bx + 2cx^2 + 3dx^3$$

$$= (3a+2b) + (2b+4c)x + (3c+6d)x^2 + 4dx^3$$