Math 2568 (§75) – April 7, 2017 Full Name: ______________________

Quiz 9

NOTE: Answers without proper justification will receive NO credit

Problem 1. *(3 points)* Use row operations to reduce the matrix $A = \begin{bmatrix} 0 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & 4 & 1 \end{bmatrix}$ to a lower-triangular matrix, and compute $\det(A)$.

Keep track of the row operations:

$$\underset{\det A}{\begin{bmatrix} 0 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & 4 & 1 \end{bmatrix}} \xrightarrow[R_1 \leftrightarrow R_2]{} \underset{-\det A}{\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 3 & 4 & 1 \end{bmatrix}} \xrightarrow[R_3 \to R_3 - 3R_1]{} \underset{-\det A}{\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & -2 & -2 \end{bmatrix}} \xrightarrow[R_3 \to R_3 + 2R_2]{} \underset{-\det A}{\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 4 \end{bmatrix}} = T$$

Then $-\det A = \det T = 1 \cdot 1 \cdot 4$, so $\boxed{\det A = -4}$

Problem 2. *(2 points)* Use Cramer's rule to solve the system $\begin{cases} x_1 + x_2 = 3 \\ x_1 - x_2 = -1 \end{cases}$

By Cramer's Rule. $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ is invertible $\det A = 1(-1) - 1 \cdot 1 = -2 \neq 0$ ✓

$B_1 = \begin{bmatrix} 3 & 1 \\ -1 & -1 \end{bmatrix} \to \det B_1 = -3 - (-1) = -2$

$B_2 = \begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix} \to \det B_2 = -1 - 3 = -4$

So $x_1 = \dfrac{\det B_1}{\det A} = \dfrac{-2}{-2} = 1$ & $x_2 = \dfrac{\det B_2}{\det A} = \dfrac{-4}{-2} = +2$.

Check: $A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ +2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1-2 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ ✓

$\boxed{\text{Solution: } \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}}$