Math 2568 (§75) – April 14, 2017 Full Name: ______________________

Quiz 10

NOTE: Answers without proper justification will receive NO credit

Problem 1. *(5 points)* Find the eigenvalues and the eigenvectors for the matrix $A = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}$.

We compute the characteristic polynomial:

$$P_A(t) = \det(A - tI_2) = \det\begin{pmatrix} 1-t & -1 \\ 1 & 3-t \end{pmatrix} = (1-t)(3-t)+1$$

$$= t^2 - 4t + 3 + 1 = t^2 - 4t + 4 = (t-2)^2$$

So $\boxed{\lambda = 2}$ is the unique eigenvalue of A.

. The eigenvectors are the solutions to:

$$\tilde{A}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{for } \tilde{A} = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$$

Use Gauss-Jordan elimination:

$$\begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \xrightarrow[R_1 \to R_1 + R_2]{} \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \quad \text{so} \quad x_1 + x_2 = 0, \quad \boxed{x_1 = -x_2}$$

The eigenvectors are $\underline{x} = \begin{bmatrix} -x_2 \\ x_2 \end{bmatrix} = x_2\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ for any x_2 in $\mathbb{R}$

We conclude: $\boxed{E_2 = \operatorname{Span}\left(\begin{bmatrix} -1 \\ 1 \end{bmatrix}\right)}$