Math 2568 (§75) – April 21, 2017 Full Name: ______________________________

Quiz 11

NOTE: Answers without proper justification will receive NO credit

Problem 1. *(5 points)* Find the eigenvalues and the eigenvectors for the matrix $A = \begin{bmatrix} 6 & 8 \\ -1 & 2 \end{bmatrix}$.

We compute $P_A(t) = \det \begin{bmatrix} 6-t & 8 \\ -1 & 2-t \end{bmatrix} = (6-t)(2-t)+8$

$= t^2-8t+12+8 = t^2-8t+20$

Roots: $\dfrac{-(-8) \pm \sqrt{8^2-4\cdot 20}}{2} = \dfrac{8 \pm \sqrt{-16}}{2} = \dfrac{8 \pm 4i}{2} = \boxed{4 \pm 2i}$ 2 eigenvalues

- E_{4+2i} = Solve for $\left(\begin{matrix} 6-(4+2i) & 8 \\ -1 & 2-(4+2i) \end{matrix}\right) = \begin{bmatrix} 2-2i & 8 \\ -1 & -2-2i \end{bmatrix}$

$\begin{bmatrix} 2-2i & 8 \\ -1 & -2-2i \end{bmatrix} \xrightarrow[R_1 \to R_1 + (2-2i)R_2]{} \begin{bmatrix} 0 & 0 \\ -1 & -2-2i \end{bmatrix}$ $\quad -x_1 + (-2-2i)x_2 = 0$

$x_1 = (-2-2i)\,x_2$

$(2-2i)(-2-2i) = -4-4+i(-4+4) = -8$

So $\underline{x} = \begin{bmatrix} (-2-2i)\,x_2 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} -2-2i \\ 1 \end{bmatrix}$ $\rightsquigarrow \boxed{E_{4+2i} = \operatorname{Span}\left(\begin{bmatrix} -2-2i \\ 1 \end{bmatrix}\right)}$

- E_{4-2i} = Solve for $\left(\begin{matrix} 6-(4-2i) & 8 \\ -1 & 2-(4-2i) \end{matrix}\right) = \begin{bmatrix} 2+2i & 8 \\ -1 & -2+2i \end{bmatrix}$

$\begin{bmatrix} 2+2i & 8 \\ -1 & -2+2i \end{bmatrix} \xrightarrow[R_1 \to R_1 + (2+2i)R_2]{} \begin{bmatrix} 0 & 0 \\ -1 & -2+2i \end{bmatrix}$ $\quad -x_1 + (-2+2i)x_2 = 0$

$x_1 = (-2+2i)\,x_2$

$(2+2i)(-2+2i) = (-4-4)+i(4-4) = -8$

So $\underline{x} = \begin{bmatrix} (-2+2i)\,x_2 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} -2+2i \\ 1 \end{bmatrix}$ $\rightsquigarrow E_{4-2i} = \operatorname{Span}\left(\begin{bmatrix} -2+2i \\ 1 \end{bmatrix}\right)$

__Alternatively:__ A is a real 2x2 matrix w/ complex conjugate eigenvalues so if v is in E_{4+2i}, then $\overline{v} = \begin{bmatrix} \overline{v_1} \\ \overline{v_2} \end{bmatrix}$ is in $E_{\overline{4+2i}} = E_{4-2i}$.

__Check:__ $A \begin{bmatrix} -2-2i \\ 1 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} -2-2i \\ 1 \end{bmatrix} = \begin{bmatrix} -12-12i+8 \\ 2+2i+2 \end{bmatrix} = \begin{bmatrix} -4-12i \\ 4+2i \end{bmatrix}$

$= (4+2i) \begin{bmatrix} \frac{-4-12i}{4+2i} \\ 1 \end{bmatrix}$ & $\dfrac{-4-12i}{4+2i} = \dfrac{-2-6i}{2+i}\,\dfrac{2-i}{2-i} = \dfrac{-4-6+(2-12)i}{5}$