SOLUTIONS

Math 2568 (§30) – Feb. 21, 2020   Full Name: ______________________

Quiz 2

NOTE: Answers without proper justfication will receive NO credit.

Problem 1. Given the matrix $A = \begin{bmatrix} -1 & 2 & 0 \\ 2 & -5 & 1 \end{bmatrix}$:

(i) *(2 points)* Find a basis for the null space $\mathcal{N}(A)$.

Solve the system $\begin{bmatrix} -1 & 2 & 0 \\ 2 & -5 & 1 \end{bmatrix} \xrightarrow[R_2 \to R_2+2R_1]{R_1 \to -R_1} \begin{bmatrix} 1 & -2 & 0 \\ 0 & -1 & 1 \end{bmatrix} \xrightarrow[R_1 \to R_1 - 2R_2]{R_2 \to -R_2} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \end{bmatrix}$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} +2x_3 \\ x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ Basis $= \left\{ \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \right\}$

(ii) *(1 points)* Find a basis for the row space of A.

Row vectors are l.i. $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = a \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} + b \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix} = \begin{bmatrix} -a+2b \\ 2a-5b \\ b \end{bmatrix}$ forces $b=0$ so $a=0$

These vectors span so $B = \left\{ \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix} \right\}$

(iii) *(2 points)* Compute a basis of the column space (or Range) of A.

Column space $=$ Row space $\left(\begin{bmatrix} -1 & 2 \\ 2 & -5 \\ 0 & 1 \end{bmatrix} \right)$

Do row reduction: $\begin{bmatrix} -1 & 2 \\ 2 & -5 \\ 0 & 1 \end{bmatrix} \xrightarrow[R_2 \leftrightarrow R_3]{R_1 \to -R_1} \begin{bmatrix} 1 & -2 \\ 0 & 1 \\ 2 & -5 \end{bmatrix} \xrightarrow[+R_2]{R_3 \to R_3 - 2R_1} \begin{bmatrix} 1 & -2 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \xrightarrow[R_1 \to R_1 + 2R_2]{} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

Basis $= \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$.   Range $= \mathbb{R}^2$.