

Math 5590H and 5111 - Honors Abstract Algebra I

Autumn 2024

Instructor: Prof. Maria Angelica Cueto (cueto.5@osu.edu)

COURSE INFORMATION

Websites: <https://carmen.osu.edu> ; <https://people.math.osu.edu/cueto.5/teaching/5590H/Au24>

Lectures: M-F 11:30am-12:25pm on Baker Systems Engineering (BE) 130.

Office Hours: Thursdays 2-4pm (room to be determined).

Course description: This is part one of a very intense year-long course on Abstract Algebra. There will be lectures five days a week, with occasional days for problem solving session, if needed.

Prerequisites: Grade C or better in Honors Linear Algebra and Differential Equations (Math 5520H), or permission of the Department.

Content: The course will be divided in three parts. In the first part, we will study Groups and their algebraic structures. In the second part, we will focus on Ring Theory, with emphasis on the commutative setting. Finally, in the last part, we will discuss Polynomial Rings as our main example, as well as various applications to Elementary Number Theory.

- *Group Theory:* Definitions, examples, basic properties. Subgroups, cosets; Lagrange's theorem. Normal subgroups, quotient groups. Simple groups. Subnormal, normal, and composition series. Conjugate elements, conjugacy classes. Normalizers and centralizers. Group homomorphisms. The isomorphism theorems. Actions of groups, orbits and stabilizers; actions of groups on themselves. Direct products of groups. Classification of finitely generated abelian groups. Groups of automorphisms. Semidirect products, permutation group, simple groups, simplicity of alternating groups. Sylow Theorems. Groups of orders p, p^2, p^3, pq, p^2q . Commutator calculus, composition series, Jordan-Hölder theorem, nilpotent and solvable groups.
- *Ring Theory:* Definitions, examples, basic properties. Rings of fractions. Ring homomorphisms, ideals, quotient rings. Isomorphism theorems. Prime and maximal ideals. Nilradical, primary ideals. Direct products of rings, the Chinese remainder theorem. Prime and irreducible elements. Noetherian and Artinian rings, Euclidean, principal ideal, unique factorization domains. Quadratic integer rings.
- *Polynomial Rings:* Roots of polynomials. Polynomials over UFDs and Gauss's lemma. Irreducibility criteria. Symmetric polynomials. Polynomials in several variables, Gröbner bases (if time allows).

Textbook D.S. Dummit and R.M. Foote, *Abstract Algebra*, 3rd edition, Wiley. We will cover Chapters 1-9 and 15 (groups, rings, polynomials) this semester. Chapters 10-14 (modules, fields, the Galois theory) will be the focus of next semester. I will mainly follow the book when preparing my lecture notes, but will sometimes change the order in which the material is presented, and skip or add some topics.

Grading. The grade will be based on homework assignments (15%), three midterms (15% each), and the final exam (40%).

- *Homework.* Homework is an essential component of this course. Its goal is to *help you understand* the material as well as to develop mathematical skills. It is thus imperative that you start working on it as soon as it is assigned, and seek help if you are stuck on a problem.

Homework will be assigned on a weekly basis (typically due on Fridays, with few exceptions indicated on the schedule at the end of this document.) There will be a list of about 4-5 problems from the textbook to submit, and a second list for practice and discussion. **Late homework will not be accepted without medical excuse.**

Assignments will be submitted in person, during class type (either handwritten solutions or type-set in L^AT_EX). Being able to communicate your ideas clearly and effectively in writing is an important component of your education. To emphasize this, *each homework will be graded for both correctness and clarity of presentation.* **The lowest homework score will be dropped.**

You are encouraged to discuss the problems with the instructors and your classmates, but your write ups must be your own. Remember: the homework is intended to help you learn the material. *Solutions can be found online, but you will learn very little if you do not think about the problems yourself first.* Do your best to *think hard about the exercises on your own before you ask for help.* If you use other people's ideas, including from an online source, you *must state this explicitly.* Failure to do so is considered academic misconduct and plagiarism, which is a serious offense.

- *Exams.* There will be three midterm exams and a final exam. Dates of these exams are included in the Course Schedule (at the end of this document). All exams will be closed-book.
- *Class Participation and Attendance.* Doing math is a human activity. We will cover the material in an interactive fashion each lecture. It is important to stay actively engaged with the material and connected with both instructors and classmates, e.g. by attending lectures.

Lectures will be approached as active learning sessions, in particular, through occasional discussions in small groups. To accommodate special situations lecture notes will be posted on the course's website. Reading of the material from the textbook ahead of time is encouraged, but not required. I expect students to attend class. Frequent absences are likely to be noted and may factor into the grade in borderline cases.

GENERAL POLICIES

Academic Misconduct Statement: It is the responsibility of the Committee on Academic Misconduct to investigate or establish procedures for the investigation of all reported cases of student academic misconduct. The term academic misconduct includes all forms of student academic misconduct wherever committed; illustrated by, but not limited to, cases of plagiarism and dishonest practices in connection with examinations. Instructors shall report all instances of alleged academic misconduct to the committee (Faculty Rule 3335-5-48.7). For additional information, see the Code of Student Conduct at <http://studentlife.osu.edu/csc/>.

Statement on Title IX: Title IX makes it clear that violence and harassment based on sex and gender are Civil Rights offenses subject to the same kinds of accountability and the same kinds of support applied to offenses against other protected categories (e.g., race). If you or someone you know has been sexually harassed or assaulted, you may find the appropriate resources at <http://titleix.osu.edu> or by contacting the Ohio State Title IX Coordinator, Kellie Brennan, at titleix@osu.edu

Your mental health: As a student you may experience a range of issues that can cause barriers to learning such as strained relationships, increased anxiety, alcohol/drug problems, feeling down, difficulty concentrating and/or lack of motivation. These mental health concerns or stressful events may lead to diminished

academic performance or reduce a student's ability to participate in daily activities. The Ohio State University offers services to assist you with addressing these and other concerns you may be experiencing. If you or someone you know are suffering from any of the aforementioned conditions, you can learn more about the broad range of confidential mental health services available on campus via the Office of Student Life's Counseling and Consultation Service (CCS) by visiting <https://ccs.osu.edu> or calling 614-292-5766. CCS is located on the 4th Floor of the Younkin Success Center and 10th Floor of Lincoln Tower. You can reach an on-call counselor when CCS is closed at 614-292-5766. Emergency help is also available through the 24/7 National Suicide Prevention Hotline at 1-800-273-TALK or at <https://suicidepreventionlifeline.org>

Respiratory Virus Guidance: The Centers for Disease Control and Prevention released new guidance (link is external) for people who test positive for COVID-19. The CDC no longer recommends a five-day isolation for those infected with the virus. Instead, individuals can leave home after they have been fever-free for at least 24 hours and have mild and improving symptoms. This move aligns COVID-19 guidance with other common respiratory viral illnesses, such as the flu.

Stay home and away from others if you are experiencing symptoms of a respiratory virus, including fever, chills, fatigue, cough, runny nose and headache.

You may return to normal activities when both are true for at least 24 hours:

- Symptoms are improving.
- Fever-free without using fever-reducing medication

In particular, please do not come to class if you believe you might have a respiratory illness.

Disability Statement: Disability Statement: The university strives to maintain a healthy and accessible environment to support student learning in and out of the classroom. If you anticipate or experience academic barriers based on your disability (including mental health, chronic, or temporary medical conditions), please let me know immediately so that we can privately discuss options. To establish reasonable accommodations, I may request that you register with Student Life Disability Services. After registration, make arrangements with me as soon as possible to discuss your accommodations so that they may be implemented in a timely fashion.

If you are ill and need to miss class, including if you are staying home and away from others while experiencing symptoms of a viral infection or fever, please let me know immediately. In cases where illness interacts with an underlying medical condition, please consult with Student Life Disability Services to request reasonable accommodations. You can connect with them at slds@osu.edu; 614-292-3307; <http://www.ods.slds.edu/>.

COURSE SCHEDULE

The following schedule is tentative only. You will be notified of any changes by email, or in class. The most recent version of this syllabus will remain available at the course's webpage. The section numbers refer to those on the course's textbook.

Week	Topics	Sections	HW
1 8/20-23	Definition of a group. Examples. Subgroups, order, generators. Cyclic groups. Free groups.	1.1–5, 6.3 2.1,2.3-4	HW1 due on 8/30
2 8/26-30	Cosets, index, normal subgroups, kernel, quotient groups. Group homomorphisms, Isomorphism theorems. Presentation.	1.6, 3.1–3 2.5	HW2 due on 9/6
September 2 - Labor day, no class			
3 9/2-6	Group actions. Orbits and stabilizers. Counting lemmas. Examples and applications.	1.7, 2.2, 4.1–3	HW3 due on 9/13
4 9/9-13	Sylow theorems - applications: groups of small order	4.5, 5.3 6.2	No homework
Midterm 1 on Tuesday 9/17			
5 9/16-20	Simple groups, Alternating groups. Direct products. Structure theorem of finite abelian groups.	3.5, 4.6 5.1–2	HW4 due on 9/27
6 9/23-27	Semidirect products, automorphism of abelian groups	4.4, 5.4–5	HW5 due on 10/4
7 9/30-10/4	Commutator subgroups, Solvable/nilpotent groups, central series, Jordan-Hölder series	6.1, 3.4	HW6 due on 10/9 Wednesday
Autumn break - October 10–11. No classes.			
8 10/7-11	Rings: definitions and examples, ring homomorphisms.	7.1-3	No homework
Midterm 2 on Friday 10/18			
9 10/14-18	Quotient Rings, Isomorphism theorems, Integral domains. Characteristic of a Ring.	7.3–4	HW7 due on 10/25
10 10/21-25	Prime and maximal ideals. Chinese remainder thm. Rings of fractions. Local rings.	7.5–6, 15.4	HW8 due on 11/1
11 10/28-11/1	Euclidean, Principal ideal and Unique factorization domains. Quadratic rings of integers	8.1–3	HW9 due on 11/8
November 5 - Election day, asynchronous learning arrangement			
12 11/4-8	Polynomial Rings. Noetherian and Artinian rings	9.1–2, 16.1 15.1–2	HW10 due on 11/15
November 11 - Veteran's day, no class			
13 11/11-15	Gauss' lemma. Eisenstein criterion of irreducibility.	9.3–4	No homework
Midterm 3 on Tuesday November 19			
14 11/18-22	Hilbert basis theorem. Groebner bases.	9.5–6	No homework
Thanksgiving break - November 27–29. No classes			
15 11/25-29	More on Groebner bases	9.6	HW11 due on 12/3 Tuesday
16 12/2-4	Review		No homework
Final exam: Thursday 12/12. 10-11.45AM.			