Math 6111 - Abstract Algebra I

Autumn 2020

Instructor: Prof. Maria Angelica Cueto

Recitations: Dr. Sjuvon Chung

Contact information: All electronic communications with the instructor and TA for this course should be done via the messaging tools on Carmen. We kindly request you not to email the instructors directly.

Course information

Websites: https://carmen.osu.edu ; https://people.math.osu.edu/cueto.5/teaching/6111/Au20

Lectures: M-W-F 11:30am-12:25pm on Zoom.

Recitations: T-R 11:30am-12:25pm on Zoom.

Office Hours: TBD (and by appointment), held online via Zoom. Questions can also be posted on Carmen's discussion boards.

Course description: This is part one of a year-long graduate course on Abstract Agebra. It covers the basic concepts of algebra that are necessary for graduate level core mathematics. It is now part of the qualifying exam system and can be used to cover the PhD Breadth Requirement for Algebra. To receive credit for the course as part of the qualifying exam structure, you must obtain an A- or higher grade.

Content: The course will be divided in three parts. In the first part, we will study Groups and their algebraic structures. In the second part, we will focus on Ring Theory, with emphasis on the commutative setting. Finally, in the last part, we will discuss various topics of linear and multilinear algebra, including tensor and exterior algebras. Time permitted, additional topics will be covered.

The following topics are considered as the syllabus for the qualifying exam.

- *Group Theory:* Monoids, subgroups, cyclic groups, homomorphisms, normal subgroups, factor groups, direct products, group actions, counting lemmas, Sylow theorems, composition series, Jordan-Hölder theorem, solvable and nilpotent groups, semi-direct products, permutation group, simple grups, simplicity of alternating groups.

- *Ring Theory:* Basic definitions of rings, ideals and modules, ring homomorphisms, commutative rings, integral domains, Chinese remainder theorem, prime and maximal ideals, prime avoidance, rings and modules of fractions, localization, Noetherian and Artinian rings, Hilbert basis theorem, Primary decomposition, PID, UFD, polynomial rings over UFDs, irreducibility criteria, symmetric polynomials, discriminant, modules over PIDs.

- *Linear and Multilinear Algebra:* Vector spaces, basic operations on vector spaces (direct sum, tensor product, dual vector spaces, Hom-spaces), bilinear and multilinear forms, quadratic forms, positive definite and semidefinite forms, tensor algebra, symmetric and exterior algebra, definition of determinant and minors of an endomorphism, automorphisms preserving a bilinear form (symplectic and orthogonal matrices), polar, Gauss and Jordan decompositions of a matrix.

Grading. The grade will be based on homework assignments (25%), two midterms (20% each), and the final exam (35%).

- *Homework.* Homework is an essential component of this course. Its goal is to help you understand the material as well as to develop mathematical skills. It is thus imperative that you start working on it as soon as it is assigned, and seek help (from the instructors and your classmates) if you are stuck on a problem.

  Homework will be assigned on a weekly basis (typically due on Fridays, with few exceptions indicated on the schedule at the end of this document.) Assignments will be submitted electronically via Carmen in pdf format (typeset in LaTeX). Each homework will be graded for correctness and clarity of presentation. The best solutions will be posted (with approval by each author) on the course's website. **Late homework will not be accepted without medical excuse**.

  You are strongly encouraged to discuss the problems with me and your classmates, but your write ups must be your own. If you use other people's ideas, including from an online source, you must state this explicitly. Active participation on Carmen discussion forum will contribute towards your homework grade.

- *Exams.* There will be three exams (all held remotely): two midterms and a final exam. Dates of these exams are included in the Course Schedule (at the end of this document). All exams will be closed-book. The final exam will have a format similar to the algebra qualifying.

- *Class Participation and Attendance.* Doing math is a human activity. We will cover the material in an interactive fashion each lecture. Although this course will be conducted entirely online, it is important to stay actively engaged with the material and connected with both instructors and classmates, e.g. by using Carmen's discussion board.

  Online lectures will be approached as active learning sessions, in particular, through discussions in small groups. Lectures will be recorded to accommodate special situations, but I expect students to attend the lectures while they are being delivered. Frequent absences are likely to be noted and may factor into the grade in borderline cases.

Prerequisites: Master's level Algebra II (Math 5112), or Grad standing. We will keep the prerequisites to a minimum, reviewing material as needed.

Suggested texts

The course will be based entirely on lecture notes, which will be posted after each class. Slides of each lecture will also be posted on the course's website.

There are a lot of excellent textbooks on abstract algebra. The following list of recommended texts is to provide some (optional) reading suggestions, and more examples/problems to work out:

- N. Jacobson, *Basic Algebra I, II*, Dover.
- S. Lang, *Algebra*, 3^{rd} edition, Springer.
- N. Bourbaki, *Algebra Chapters 1-3*, Springer.
- D.S. Dummit and R.M. Foote, *Abstract Algebra*, 3^{rd} edition, Wiley.
- M.F. Atiyah and G. MacDonald,*Introduction to Commutative Algebra*, Addison Wesley.
- D. Eisenbud, *Commutative Algebra: with a view toward Algebraic Geometry*, Springer.

Course technology

The course will be delivered entirely online. You should be able to connect to CarmenZoom with audio, video and chat participation. Course annoncements will be made regularly through Carmen. **It is strongly encouraged that you connect to Carmen regularly (at least three times a week).**

For help with your password, university e-mail, Carmen, or any other technology issues, questions, or requests, contact the OSU IT Service Desk. Standard support hours are available at

https://ocio.osu.edu/help/hours.

Support for urgent issues is available 24x7.

- *Self-Service and Chat support:* http://ocio.osu.edu/selfservice
- *Phone:* 614-688-HELP (4357)
- *Email:* 8help@osu.edu
- *TDD:* 614-688-8743

General policies

Academic Misconduct Statement: It is the responsibility of the Committee on Academic Misconduct to investigate or establish procedures for the investigation of all reported cases of student academic misconduct. The term academic misconduct includes all forms of student academic misconduct wherever committed; illustrated by, but not limited to, cases of plagiarism and dishonest practices in connection with examinations. Instructors shall report all instances of alleged academic misconduct to the committee (Faculty Rule 3335-5-48.7). For additional information, see the Code of Student Conduct at http://studentlife.osu.edu/csc/.

Statement on Title IX: Title IX makes it clear that violence and harassment based on sex and gender are Civil Rights offenses subject to the same kinds of accountability and the same kinds of support applied to offenses against other protected categories (e.g., race). If you or someone you know has been sexually harassed or assaulted, you may find the appropriate resources at http://titleix.osu.edu or by contacting the Ohio State Title IX Coordinator, Kellie Brennan, at titleix@osu.edu

Disability Statement: The University strives to make all learning experiences as accessible as possible. If you anticipate or experience academic barriers based on your disability (including mental health, chronic or temporary medical conditions), please let me know immediately so that we can privately discuss options. To establish reasonable accommodations, I may request that you register with Student Life Disability Services. After registration, make arrangements with me as soon as possible to discuss your accommodations so that they may be implemented in a timely fashion. SLDS contact information: slds@osu.edu; 614-292-3307; http://www.ods.osu.edu/; 098 Baker Hall, 113 W. 12th Avenue.

Your mental health: As a student you may experience a range of issues that can cause barriers to learning such as strained relationships, increased anxiety, alcohol/drug problems, feeling down, difficulty concentrating and/or lack of motivation. These mental health concerns or stressful events may lead to diminished academic performance or reduce a students ability to participate in daily activities. The Ohio State University offers services to assist you with addressing these and other concerns you may be experiencing. If you or someone you know are suffering from any of the aforementioned conditions, you can learn more about the broad range of confidential mental health services available on campus via the Office of Student Lifes Counseling and Consultation Service (CCS) by visiting https://ccs.osu.edu or calling 614- 292-5766. CCS is located on the 4th Floor of the Younkin Success Center and 10th Floor of Lincoln Tower. You can reach an on-call counselor when CCS is closed at 614-292-5766. Emergency help is also available through the 24/7 National Suicide Prevention Hotline at 1-800-273- TALK or at https://suicidepreventionlifeline.org

Course Schedule

The following schedule is tentative only. You will be notified of any changes by email, or in class. The most recent version of this syllabus will remain available at the course's webpage.

Week	Topics	Homework deadline
1 8/25-28	Definitions (Groups, homomorphisms, normal subgroups). Presentation of a group. Groups actions. Counting lemmas.	HW 1 due on 9/4
2 8/31-9/4	Quotient groups. Simple groups. Basic Isomorphism Theorems	HW 2 due on 9/11
September 7 - Labour day, no class		
3 9/7-9/11	Short exact sequences. Semidirect products. Sylow Theorems.	HW 3 due on 9/18
4 9/14-18	Composition series. Jordan-Hölder theorem Derived groups. Solvable and nilpotent groups.	HW 4 due on 9/25
5 9/21-25	Symmetric groups. Simplicity of alternating groups.	HW 5 due on 9/30 **Wednesday**
First midterm on 10/2 (Friday)		
6 9/28-10/2	Definitions (rings, ideals). Isomorphism theorems. Characteristic. Modules - direct sums, products, homomorphisms, exact sequences.	HW 6 due on 10/9
7 10/5-9	Commutative rings. Prime and maximal ideals. Integral domains. Rings of fractions. Prime avoidance, Chinese Remainder Theorem.	HW 7 due on 10/16
8 10/12-16	Localization, Artinian and Noetherian rings. Hilbert Basis Theorem, Nilpotent and Jacobson radicals.	HW 8 due on 10/23
9 10/19-23	Primary decomposition, PIDs, modules over PIDs.	HW 9 due on 10/30
10 10/26-30	Euclidean domains, PID and UFD. Gauss lemma. Eisenstein criterion for irreducibility.	HW 10 due on 11/06
11 11/2-6	Symmetric polynomials, discriminants of polynomials. Valuation rings and DVRs.	No homework
Second midterm on 11/9 (Monday).		
November 11 - Veteran's day, no class		
12 11/9-13	Vector spaces. Basic operations on vector spaces: direct sum, tensor product, dual vector spaces, Hom-spaces.	HW 11 due on 11/16 **Monday**
13 11/16-20	Bilinear and multilinear forms. Quadratic forms. Positive (semi)definite forms. Symplectic forms.	HW 12 due on 11/25 **Wednesday**
14 11/23-27	Tensor algebra. Symmetric and Exterior algebra. Determinants and minors of endomorphisms, Cayley-Hamilton theorem.	HW 13 due on 12/3 **Thursday**
November 26-27 - Thanksgiving break, no classes		
15 11/30-12/4	Automorphisms preserving a bilinear form: symplectic and orthogonal matrices. Decomposition theorems: polar, Jordan and Gaussian.	No homework

Final exam: Friday 12/11. 10-11.45AM.