

Math 6111 - Abstract Algebra I

Autumn 2021

Instructor: Prof. Maria Angelica Cueto (cueto.5@osu.edu)

Recitations: Dr. Soumya Sankar (sankar.40@osu.edu)

COURSE INFORMATION

Websites: <https://carmen.osu.edu> ; <https://people.math.osu.edu/cueto.5/teaching/6111/Au21>

Lectures: M-W-F 11:30am-12:25pm on Derby Hall 048.

Recitations: T-R 11:30am-12:25pm on Derby Hall 048.

Office Hours: By appointment, held online via Zoom. Questions can also be posted on Carmen's discussion boards.

Course description: This is part one of a year-long graduate course on Abstract Algebra. It covers the basic concepts of algebra that are necessary for graduate level core mathematics. It is now part of the qualifying exam system and can be used to cover the PhD Breadth Requirement for Algebra. To receive credit for the course as part of the qualifying exam structure, you must obtain an A- or higher grade.

Content: The course will be divided in three parts. In the first part, we will study Groups and their algebraic structures. In the second part, we will focus on Ring Theory, with emphasis on the commutative setting. Finally, in the last part, we will discuss various topics of linear and multilinear algebra, including tensor and exterior algebras. Time permitted, additional topics will be covered.

The following topics are considered as the syllabus for the qualifying exam.

- *Group Theory:* Monoids, subgroups, cyclic groups, homomorphisms, normal subgroups, factor groups, direct products, group actions, counting lemmas, Sylow theorems, composition series, Jordan-Hölder theorem, solvable and nilpotent groups, semi-direct products, permutation group, simple groups, simplicity of alternating groups.
- *Ring Theory:* Basic definitions of rings, ideals and modules, ring homomorphisms, commutative rings, integral domains, Chinese remainder theorem, prime and maximal ideals, prime avoidance, rings and modules of fractions, localization, Noetherian and Artinian rings, Hilbert basis theorem, Primary decomposition, PID, UFD, polynomial rings over UFDs, irreducibility criteria, symmetric polynomials, discriminant, modules over PIDs.
- *Linear and Multilinear Algebra:* Vector spaces, basic operations on vector spaces (direct sum, tensor product, dual vector spaces, Hom-spaces), bilinear and multilinear forms, quadratic forms, positive definite and semidefinite forms, tensor algebra, symmetric and exterior algebra, definition of determinant and minors of an endomorphism, automorphisms preserving a bilinear form (symplectic and orthogonal matrices), polar, Gauss and Jordan decompositions of a matrix.

Grading. The grade will be based on homework assignments (20%), two midterms (25% each), and the final exam (30%).

- *Homework.* Homework is an essential component of this course. Its goal is to *help you understand* the material as well as to develop mathematical skills. It is thus imperative that you start working

on it as soon as it is assigned, and seek help if you are stuck on a problem.

Homework will be assigned on a weekly basis (typically due on Fridays, with few exceptions indicated on the schedule at the end of this document.) Assignments will be submitted electronically via Carmen in pdf format (either typeset in L^AT_EX or scan of handwritten solutions). Being able to communicate your ideas clearly and effectively in writing is an important component of your graduate education. To emphasize this, *each homework will be graded for both correctness and clarity of presentation*. The best solutions will be posted (with approval by each author) on the course's website. **Late homework will not be accepted without medical excuse.**

You are encouraged to discuss the problems with the instructors and your classmates, but your write ups must be your own. Remember: the homework is intended to help you learn the material. Do your best to *first think hard about the exercises on your own before you ask for help*. If you use other people's ideas, including from an online source, you *must state this explicitly*. Active participation on Carmen discussion forum will contribute towards your homework grade.

- *Exams*. There will be three exams: two midterms and a final exam. Dates of these exams are included in the Course Schedule (at the end of this document). All exams will be closed-book. The final exam will have a format similar to the algebra qualifying.
- *Class Participation and Attendance*. Doing math is a human activity. We will cover the material in an interactive fashion each lecture. It is important to stay actively engaged with the material and connected with both instructors and classmates, e.g. by attending lectures and using Carmen's discussion board.

Lectures will be approached as active learning sessions, in particular, through discussions in small groups. To accommodate special situations (e.g. isolation/quarantine due to COVID-19 exposure or positive test), lecture notes will be posted on the course's website. I expect students to attend both lectures and recitations. Frequent absences are likely to be noted and may factor into the grade in borderline cases.

Prerequisites: Master's level Algebra II (Math 5112), or Grad standing. We will keep the prerequisites to a minimum, reviewing material as needed.

SUGGESTED TEXTS

The course will be based entirely on lecture notes, which will be posted after each class. There are a lot of excellent textbooks on abstract algebra. The following list of recommended texts is to provide some (optional) reading suggestions, and more examples/problems to work out:

- N. Jacobson, *Basic Algebra I, II*, Dover.
- S. Lang, *Algebra*, 3rd edition, Springer.
- N. Bourbaki, *Algebra Chapters 1-3*, Springer.
- D.S. Dummit and R.M. Foote, *Abstract Algebra*, 3rd edition, Wiley.
- M.F. Atiyah and G. MacDonald, *Introduction to Commutative Algebra*, Addison Wesley.
- D. Eisenbud, *Commutative Algebra: with a view toward Algebraic Geometry*, Springer.

HEALTH AND SAFETY POLICIES

All students, faculty and staff are required to comply with and stay up to date on all university safety and health guidance (see <https://safeandhealthy.osu.edu>), which includes following university mask policies. Non-compliance will be warned first and disciplinary actions will be taken for repeated offenses.

Everyone is strongly encouraged to get vaccinated against COVID-19 as soon as they are able to do so. The vaccine has been proven to be safe and effective. The university has and will always base decisions on evidence-based public health guidance and the state of the pandemic, and a high vaccination rate will only increase the chances of normality.

Unvaccinated students will be required to test weekly as the academic year begins. Students who have been vaccinated against COVID-19 and reported their status to the university will not have to test. Once [House Bill 244](#) goes into effect on October 14, all students will be tested with the same protocols, regardless of vaccination status.

Regardless of vaccination status, masks must be worn **at all times** during in-person classes (unless otherwise indicated by University policy). Social distancing will be encouraged, whenever possible. Face masks must be worn in indoor settings, including, but not limited to: classrooms, residence halls, conference rooms, shared office spaces, hallways, buses, shared vehicles and common areas such as lobbies, study spaces and laundry facilities on all Ohio State campuses.

Free face masks are available for students, faculty and staff at a variety of campus locations including: all recreation facilities, most university libraries and the Ohio Union information desk. Masks can be picked up during business hours. A list of pick-up locations around campus can be found at

<https://safeandhealthy.osu.edu/personal-protection-hygiene>

ADDITIONAL RESOURCES

The Ohio State University Wexner Medical Center's [Coronavirus Outbreak site](#) includes the latest information about COVID-19 as well as guidance for students, faculty and staff.

The [Office of Student Life's 'We Are Here For You'](#) page includes several resources to support your mental, physical and financial health.

COURSE TECHNOLOGY

Course announcements will be made regularly through Carmen. **It is strongly encouraged that you connect to Carmen regularly (at least three times a week).**

For help with your password, university e-mail, Carmen, or any other technology issues, questions, or requests, contact the OSU IT Service Desk. Standard support hours are available at

<https://ocio.osu.edu/help/hours>.

Support for urgent issues is available 24x7.

- *Self-Service and Chat support:* <http://ocio.osu.edu/selfservice>
- *Email:* 8help@osu.edu; *Phone:* 614-688-HELP (4357); *TDD:* 614-688-8743.

GENERAL POLICIES

Academic Misconduct Statement: It is the responsibility of the Committee on Academic Misconduct to investigate or establish procedures for the investigation of all reported cases of student academic misconduct. The term academic misconduct includes all forms of student academic misconduct wherever committed; illustrated by, but not limited to, cases of plagiarism and dishonest practices in connection with examinations. Instructors shall report all instances of alleged academic misconduct to the committee (Faculty Rule 3335-5-48.7). For additional information, see the Code of Student Conduct at <http://studentlife.osu.edu/csc/>.

Statement on Title IX: Title IX makes it clear that violence and harassment based on sex and gender are Civil Rights offenses subject to the same kinds of accountability and the same kinds of support applied to offenses against other protected categories (e.g., race). If you or someone you know has been sexually harassed or assaulted, you may find the appropriate resources at <http://titleix.osu.edu> or by contacting the Ohio State Title IX Coordinator, Kellie Brennan, at titleix@osu.edu

Disability Statement: The university strives to make all learning experiences as accessible as possible. In light of the current pandemic, students seeking to request COVID-related accommodations may do so through the university's request process, managed by Student Life Disability Services. More information can be found on

<https://slds.osu.edu/covid-19-info/covid-related-accommodation-requests/>

If you anticipate or experience academic barriers based on your disability (including mental health, chronic, or temporary medical conditions), please let me know immediately so that we can privately discuss options. To establish reasonable accommodations, I may request that you register with Student Life Disability Services. After registration, make arrangements with me as soon as possible to discuss your accommodations so that they may be implemented in a timely fashion. SLDS contact information: slds@osu.edu; 614-292-3307; <http://www.ods.osu.edu/>; 098 Baker Hall, 113 W. 12th Ave.

Your mental health: As a student you may experience a range of issues that can cause barriers to learning such as strained relationships, increased anxiety, alcohol/drug problems, feeling down, difficulty concentrating and/or lack of motivation. These mental health concerns or stressful events may lead to diminished academic performance or reduce a student's ability to participate in daily activities. The Ohio State University offers services to assist you with addressing these and other concerns you may be experiencing. If you or someone you know are suffering from any of the aforementioned conditions, you can learn more about the broad range of confidential mental health services available on campus via the Office of Student Life's Counseling and Consultation Service (CCS) by visiting <https://ccs.osu.edu> or calling 614-292-5766. CCS is located on the 4th Floor of the Younkin Success Center and 10th Floor of Lincoln Tower. You can reach an on-call counselor when CCS is closed at 614-292-5766. Emergency help is also available through the 24/7 National Suicide Prevention Hotline at 1-800-273-TALK or at <https://suicidepreventionlifeline.org>

COURSE SCHEDULE

The following schedule is tentative only. You will be notified of any changes by email, or in class. The most recent version of this syllabus will remain available at the course's webpage.

Week	Topics	Homework deadline
1 8/24-27	Definitions (Groups, homomorphisms, normal subgroups). Quotient groups. Basic Isomorphism Theorems.	HW 1 due on 9/3
2 8/30-9/3	Presentation of a group. Groups actions. Counting lemmas.	HW 2 due on 9/10
September 6 - Labour day, no class		
3 9/6-9/10	Sylow Theorems. Simple groups.	HW 3 due on 9/17
4 9/13-17	Semidirect products. Short exact sequences. Composition series.	HW 4 due on 9/24
5 9/20-24	Jordan-Hölder theorem. Derived groups. Solvable and nilpotent groups. Simplicity of alternating groups.	HW 5 due on 9/29 Wednesday
First midterm on 10/1 (Friday)		
6 9/27-10/1	Definitions (rings, ideals). Isomorphism theorems. Characteristic. Modules - direct sums, products, homomorphisms, exact sequences.	HW 6 due on 10/08
7 10/4-8	Commutative rings. Prime and maximal ideals. Integral domains. Rings of fractions. Prime avoidance, Chinese Remainder Theorem.	HW 7 due on 10/15
October 14-15 - Autumn Break, no class.		
8 10/11-15	Localization, Noetherian rings, Hilbert Basis Theorem, Nilpotent and Jacobson radicals.	HW 8 due on 10/22
9 10/18-22	Artinian rings, Hensel's lemma, Primary decomposition	HW 9 due on 10/29
10 10/25-29	Free modules, torsion modules, PIDs, modules over PIDs.	HW 10 due on 11/05
11 11/1-5	Modules over PIDS, Smith normal forms, Rational normal forms Jordan normal forms of matrices	No homework
Second midterm on 11/12 (Friday).		
November 11 - Veteran's day, no class		
12 11/8-12	Nakayama's Lemma, Cayley-Hamilton theorem. Vector spaces. Basic operations: direct sum, dual vector spaces, Hom-spaces.	HW 11 due on 11/19
13 11/15-19	Bilinear and multilinear forms. Tensor products. Hom-tensor adjointness.	HW 12 due on 12/1 Wednesday
November 24-26 - Thanksgiving break, no classes		
14 11/22-26	Canonical tensors. Tensor, Symmetric and Exterior algebra.	No homework
15 11/29-12/3	Determinants and minors of endomorphisms. Gaussian decomposition. Bilinear, symmetric and skew-symmetric forms.	HW 13 due on 12/8 Wednesday
16 12/6-8	Classification of bilinear real symmetric and skew-symmetric forms. Automorphisms preserving bilinear forms. Polar decomposition.	No homework

Final exam: Thursday 12/16, 10-11.45AM (tentative date).