Math 6501 - Enumerative Combinatorics I

Autumn 2019

Instructor: Prof. Maria Angelica Cueto

Office Hours: M-W 10:30am-12:00pm in MW 636 (and by appointment at `cueto.5@osu.edu`)

Lectures: M-W-F 3:00-3:55pm Bolz Hall (BO) 120.

Website: https://people.math.osu.edu/cueto.5/teaching/6501/Au19

Textbook: *Enumerative Combinatorics, Volume I*, Second edition, by Richard Stanley (see [6].)

Course description: This is part one of a year-long graduate course on Enumerative Combinatorics and related fields. Our main goal is to address two questions: what does it mean to count and how can we effectively do so. Emphasis will be given to classical fun examples. Highlights and connections to other fields of mathematics and applications will be discussed throughout the semester.

The course will be divided in two parts. In the first part, we will study Basic Structures of enumeration, Sieve Methods and Generating functions. In the second part, we will discuss the structure of partially ordered sets, lattices, Möbius functions, and applications of enumerative aspects of geometric combinatorics, such as hyperplane arrangements. Additional topics will include the basics of matroid theory, and the geometry of Polytopes, triangulations and Erhart theory.

Tentative list of topics: basic counting principles, generating functions, recurrence relations, lattice polytopes, Ehrhart theory, combinatorial species, tree enumeration, Sieve methods, Catalan numbers, posets, Möbius inversion, hyperplane arrangements, triangulations of polytopes, and matroids.

Tentative Schedule: The following schedule is tentative and subject to change.

Week 1: Introduction to counting and overview; Binomial coefficients and identities.
Week 2: Fundamental integer sequences; multinomial identities;
Lattice paths; q-binomial coefficients.
Week 3: Sieve methods; Inclusion-exclusion.
Week 4: The Gessel-Viennot theorem; counting trees; the Matrix-Tree theorem.
Week 5: Unimodality and log-concavity sequences; applications of Sieve Methods.
Week 6: Formal Series; Generating functions; Rational Generating Functions.
Week 7: Examples; Catalan numbers, Bernoulli numbers; compositions; products.
Week 8: Exponential generating functions; Partition identities; Combinatorial species.
Week 9: Basics of Polytopes; Triangulations; Secondary fans and secondary polytopes.
Week 10: Introduction to Ehrhart Theory.
Week 11: Posets; Lattices and their refinement; the incidence algebra of a poset.
Week 12: The zeta and Möbius functions for posets; the Möbius inversion formula.
Week 13: Rank Selection; R-labelings.
Week 14: Differential Posets; Sperner Property.
Week 15 & 16: Hyperplane arrangements; basics on Matroids.

Grading Policy: Your final raw score (and grade) for this course will be computed as follows:

Homework:	80%	In-class participation:	20%

Participation: I expect every student to **attend** all lectures and **actively participate** in in-class discussions. Doing math is a human activity. We will cover the material in an interacting fashion each lecture.

Homework: There will be regular homework assignements, roughly one every four lectures, posted on the course's website. The best solutions will be posted (with approval by each author). Using LaTeX is highly encouraged, but not mandatory.

You are strongly encouraged to discuss the problems with me and your classmates, but you **must** write up your own solutions without looking at solutions of others. If you use other people's ideas, including from an online source, you must state this explicitly.

Academic Misconduct Statement: It is the responsibility of the Committee on Academic Misconduct to investigate or establish procedures for the investigation of all reported cases of student academic misconduct. The term academic misconduct includes all forms of student academic misconduct wherever committed; illustrated by, but not limited to, cases of plagiarism and dishonest practices in connection with examinations. Instructors shall report all instances of alleged academic misconduct to the committee (Faculty Rule 3335-5-48.7). For additional information, see the Code of Student Conduct at http://studentlife.osu.edu/csc/.

Disability Statement: The University strives to make all learning experiences as accessible as possible. If you anticipate or experience academic barriers based on your disability (including mental health, chronic or temporary medical conditions), please let me know immediately so that we can privately discuss options. To establish reasonable accommodations, I may request that you register with Student Life Disability Services. After registration, make arrangements with me as soon as possible to discuss your accommodations so that they may be implemented in a timely fashion. SLDS contact information: slds@osu.edu; 614-292-3307; http://www.ods.osu.edu/; 098 Baker Hall, 113 W. 12th Avenue.

References

[1] M. Aigner. *A course in enumeration*, volume 238 of *Graduate Texts in Mathematics*. Springer, Berlin, 2007.

[2] M. Beck and S. Robins. *Computing the continuous discretely*. Undergraduate Texts in Mathematics. Springer, New York, second edition, 2015. Integer-point enumeration in polyhedra, With illustrations by David Austin.

[3] J. A. De Loera, J. Rambau, and F. Santos. *Triangulations*, volume 25 of *Algorithms and Computation in Mathematics*. Springer-Verlag, Berlin, 2010. Structures for algorithms and applications.

[4] J. Oxley. *Matroid theory*, volume 21 of *Oxford Graduate Texts in Mathematics*. Oxford University Press, Oxford, second edition, 2011.

[5] R. P. Stanley. An introduction to hyperplane arrangements. In *Geometric combinatorics*, volume 13 of *IAS/Park City Math. Ser.*, pages 389–496. Amer. Math. Soc., Providence, RI, 2007.

[6] R. P. Stanley. *Enumerative combinatorics. Volume 1*, volume 49 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, second edition, 2012.

[7] R. P. Stanley. *Algebraic combinatorics*. Undergraduate Texts in Mathematics. Springer, Cham, 2018. Walks, trees, tableaux, and more, Second edition of [MR3097651].