Math 7141 - Algebraic Geometry I

Autumn 2023

Instructor: Prof. Maria Angelica Cueto

Office Hours: F 2-3pm in MW 636 (and by appointment at `cueto.5@osu.edu`)

Lectures: M-W-F 11:30-12:25pm Smith Lab (SM) 1042.

Website: https://people.math.osu.edu/cueto.5/teaching/7141/Au23

Textbook: I will based my lectures on material available in several books:

- *Basic Algebraic Geometry (volumes 1 and 2)*, by Igor Shafarevich (Springer-Verlag, Second edition, 1994).
- *Algebraic geometry: a first course* by Joe Harris (Springer-Verlag, Graduate Texts in Mathematics Series, GTM 133, 1992).
- *Ideals, varieties, and algorithms: an introduction to computational algeraic geometry and commutative algebra* by David Cox, John Little and Donal OShea (Springer, Undergraduate texts in mathematics Series, fourth edition, 2015).
- *Algebraic Curves and Riemann Surfaces* by Rick Miranda (AMS Graduate Studies in Mathematics, Volume 5).
- *Introduction to Algebraic Curves* by Phillip A. Griffiths (AMS, Translations of mathematics Monographs, Volume 76).

Course description: This is part one of a year-long graduate course on Algebraic Geometry.

Algebraic geometry is an old and amazingly interdisciplinary and active subject, borrowing ideas from topology, differential geometry, number theory, and analysis. In this course the goal is to become acquainted with the basics, affine and projective varieties, dimension, tangent spaces, smoothness, blowups, sheaves and cohomology, algebraic curves and some of the fundamental theory that governs their geometry. Emphasis will be given to classical examples and hands-on computations, including Grassmannians, flag and determinantal varieties, Segre and Veronese maps. Time permitted, we will discuss topics on computational algebraic geometry, including an overview of Groebner bases.

By participating in this course, students will learn about this beautiful subject and will gain some insight into deep yet technical results in algebraic geometry by means of concrete hands-on examples.

Disclaimer: Algebraic geometry is a beautiful subject known for its technicality, but the reason for it lies in the extremely large range of geometric phenomena that appear in the subject. Consequently, students who are serious about the subject, should spend this course trying to see as much of this range as they possibly can. We will see a large number of examples in lectures. I recommend learning a few examples a week from Harris's "Algebraic Geometry: A First Course" to build your own geometric database.

Prerequisites: Some experience with category theory, rings and modules, multilinear algebra (at the level of Math 6112), and Commutative Algebra (Math 6151).

Tentative list of topics: affine algebraic varieties, Hilbert nullstellensatz, irreducible components, primary decompositions, Zarisky topology, Noether normalization, going-up and going-down theorems, productos of varieties, morphisms; projective varieties, homogeneous ideals; projective nullstellensatz, Hilbert polynomials, homogeneization; Grassmannians, flag varities, determinantal varieties; dimension theory, smoothness, blowups, sheaves and cohomology, divisors and line bundles; algebraic curves and their geometry, the Riemann-Roch theorem and its consequences, canonical projective morphisms, ampleness and very ampleness, criteria for embeddings, hyperelliptic curves, the Jacobian of a curve and the Abel-Jacobi map; del Pezzo surfaces, the 27 lines on a smooth cubic surface in $\mathbb{P}^3$.

References: The literature on embedded algebraic Varieties is vast. We will be using several references to cover the material discussed in class. Links to electronic copies available through the OSU library are provided whenever possible (access requires connection via an OSU proxy, e.g., by being on campus)

1. *Basic Algebraic Geometry (volumes 1 and 2)*, by Igor Shafarevich (Springer-Verlag, Second edition, 1994).
2. *Algebraic geometry: a first course* by Joe Harris (Springer-Verlag, Graduate Texts in Mathematics Series, GTM 133, 1992).
3. *Ideals, varieties, and algorithms: an introduction to computational algeraic geometry and commutative algebra* by David Cox, John Little and Donal OShea (Springer, Undergraduate texts in mathematics Series, fourth edition, 2015). Available online through the OSU library.
4. *Commutative Algebra: with a View Toward Algebraic Geometry* by David Eisenbud (Springer, Graduate Texts in Mathematics Series, GTM 150, 2004). Available online through the OSU library.
5. *Algebraic Curves and Riemann Surfaces* by Rick Miranda (AMS Graduate Studies in Mathematics, Volume 5).
6. *Introduction to Algebraic Curves* by Phillip A. Griffiths (AMS, Translations of mathematics Monographs, Volume 76).

Grading Policy: Your final raw score (and grade) for this course will be computed as follows:

Homework:	80%	In-class participation:	20%

Participation: I expect every student to **attend** all lectures and **actively participate** in in-class discussions. Doing math is a human activity. We will cover the material in an interacting fashion each lecture.

Homework: There will be regular homework assignements, roughly one every two weeks, posted on the course's website. Participants are encourage to work in teams, but *individual solutions* must be submitted for grading and credit. If you use other people's ideas, including from an online source, you must state this explicitly.

Each student is expected to work on and submit one or two problems per assignment (chosen by the students). **Only one solution per problem will be uploaded to Carmen for grading.** Students should agree among their peers which problems each of them will submit (and inform me by comments on Carmen to the assignment post).

Solutions should be uploaded as a pdf file (preferrably produced in LaTeX).

As an optional complementary assignment, students are welcome to give a 30 minute presentation in class on a topic not discussed in class. Topics will be selected in agreement with the instructor.

Religious accommodations Statement: It is Ohio States policy to reasonably accommodate the sincerely held religious beliefs and practices of all students. The policy permits a student to be absent for up to three days each academic semester for reasons of faith or religious or spiritual belief.

Students planning to use religious beliefs or practices accommodations for course requirements must inform the instructor in writing no later than 14 days after the course begins. The instructor is then responsible for scheduling an alternative time and date for the course requirement, which may be before or after the original time and date of the course requirement. These alternative accommodations will remain confidential. It is the students responsibility to ensure that all course assignments are completed.

Academic Misconduct Statement: It is the responsibility of the Committee on Academic Misconduct to investigate or establish procedures for the investigation of all reported cases of student academic misconduct. The term academic misconduct includes all forms of student academic misconduct wherever committed; illustrated by, but not limited to, cases of plagiarism and dishonest practices in connection with examinations. Instructors shall report all instances of alleged academic misconduct to the committee (Faculty Rule 3335-5-48.7). For additional information, see the Code of Student Conduct at http://studentlife.osu.edu/csc/.

Disability Statement: The University strives to make all learning experiences as accessible as possible. If you anticipate or experience academic barriers based on your disability (including mental health, chronic or temporary medical conditions), please let me know immediately so that we can privately discuss options. To establish reasonable accommodations, I may request that you register with Student Life Disability Services. After registration, make arrangements with me as soon as possible to discuss your accommodations so that they may be implemented in a timely fashion. SLDS contact information: slds@osu.edu; 614-292-3307; http://www.ods.osu.edu/; 098 Baker Hall, 113 W. 12th Avenue.

Tentative Schedule: The following schedule is tentative and subject to change.

- **Week 1:** Introduction and overview; affine algebraic varieties and first examples; ideals defining varieties; Hilbert basis theorem.
- **Week 2:** Hilbert nullstellensatz; irreducible components and primary decompositions; embedded components; the case of monomial ideals.
- **Week 3:** Noether Normalization; going-up and going down theorems.
- **Week 4:** Products, morphism between affine varieties; rational maps; examples; Zariski topology.
- **Week 5:** Projective varieties; homogeneous ideals; graded algebras; homogeneous nullstellensatz; Hilbert polynomials.
- **Week 6:** Homogeneization of affine varieties. Rational, birational, finite, flat, proper and projective morphisms; products; Segre and Veronese maps; the structure sheaf; examples: projective space, Grassmannians, flag varieties, determinantal varieties.
- **Week 7:** Sheaf theory; remarks on local rings; tangent spaces; smoothness; dimension theory; function fields; Differentials and birational invariance and dimension.
- **Week 8:** Computations in Algebraic Geometry using Groebner bases; dimension of projective varieties via Hilbert polynomials.
- **Week 9:** Blowups and resolution of singularties; resolution of curves.
- **Week 10:** Divisors and the divisor class group, line bundles; Čech cohomology.
- **Week 11:** Sheaf cohomology; examples in $\mathbb{P}^n$.
- **Week 12:** Curves and divisors on curves; differentials; Bezoút's Theorem.
- **Week 13-14:** Canonical divisors, genus of a curve; Riemann Roch Theorem and its consequeces; canonical projective morphisms, ampleness and very ampleness, criteria for embeddings; hyperelliptic curves.
- **Week 15:** The Jacobian of a curve, and the Abel-Jacobi map.
- **Week 16:** Del Pezzo surfaces; the 27 lines on a smooth cubic surface in $\mathbb{P}^3$.