Math 8140 - Topics in Algebraic Geometry (Tropical Geometry)

Spring 2017

Instructor: Maria Angelica Cueto

Office Hours: M-W-Thur 2:00pm-3:00pm in MW 620 (and by appointment at `cueto.5@osu.edu`)

Lectures: MWF 9:10-10:05AM Cockins Hall (CH) 228.

Website: https://people.math.osu.edu/cueto.5/teaching/8140/Sp17

Course description: This is a one semester graduate topics course on tropical geometry, an emerging field bridging combinatorics, algebraic geometry, and non-Archimedean geometry, with applications.

In the first part of the course, we will study "embedded" tropical varieties: given a subvariety of the algebraic n-dimensional torus over a non-Archimedean valued field. We will define a polyhedral complex in $\mathbb{R}^n$ called its tropicalization. Topics include: structure of tropical varieties, the fundamental theorem of tropical geometry, tropical linear spaces and the tropical Grassmannian, including matroid theory, tropical compactifications, toric degenerations and the tropical computation of plane relative Gromov-Witten invariants. We will loosely follow the textbook [14] for most of this topics, although certain papers will also be used as references.

In the second part of the course we will study tropicalizations from an abstract perspective and in connection with Berkovich non-Archimedean analytic spaces. We will focus on the case of abstract tropical curves, which are vertex-weighted metric graphs, and discuss their relation to algebraic and non-Archimedean analytic curves. Our goal would be to construct and study the moduli spaces of tropical curves. We will cover many cutting-edge papers in the field (see the References).

Tentative Schedule: The following schedule is tentative and subject to change.

Week 1: Introduction and first examples; fields with valuations; polyhedral complexes.
Week 2: Tropical hypersurfaces; Fundamental Theorem and Structure Theorem for hypersurfaces.
Week 3: Gröbner basis over valued fields; initial ideals of homogeneous ideals.
Week 4: the Gröbner complex; tropical basis, Fundamental Theorem in any codimension
Week 5: Matroids; Grassmannians and their matroid stratification.
Week 6: Tropical Grassmannians; Tropical linear spaces, Bergman fans of matroids.
Week 7: Multiplicity and balancing; Connectivity; Structure Theorem of tropical varieties
Week 8: Abstract tropical varieties; Tropical linear spaces (rev.); Chow varieties and polytopes.
Week 9: Toric Degenerations; Introduction to Toric varieties; Tropical Compactifications.
Week 10-11: Geometric tropicalization; Berkovich analytic spaces; limits of tropicalizations.
Week 12: Moduli of tropical curves.
Week 13: Student presentations.
Week 14: Mikhalkin's Correspondence Theorem; GW theory of elliptic complex curves.
Week 15: Tropical geometry of genus 2 curves.

Grading Policy: Your final raw score (and grade) for this course will be computed as follows:

Homework	60%
Final Report and in-class Presentation	40%

Homework: There will be 3-4 homeworks covering the material for the first 9 weeks of class.

Final Project: For the final project of this class, you will give a presentation in class and write a paper. The topic can be an exposition of a research paper related to tropical geometry. You are encouraged to think about further questions related to the paper. Only one student can work on a given paper, and projects will be assigned on a first come-first served basis. A list of topics will be posted on the course's website.

Academic Misconduct Statement: It is the responsibility of the Committee on Academic Misconduct to investigate or establish procedures for the investigation of all reported cases of student academic misconduct. The term academic misconduct includes all forms of student academic misconduct wherever committed; illustrated by, but not limited to, cases of plagiarism and dishonest practices in connection with examinations. Instructors shall report all instances of alleged academic misconduct to the committee (Faculty Rule 3335-5-48.7). For additional information, see the Code of Student Conduct at http://studentlife.osu.edu/csc/.

Disability Statement: If you have a documented disability, please register with Student Life Disability Services. After registration, make arrangements with me as soon as possible to discuss your accommodations, so they may be implemented in a timely fashion. If you have any questions about this process, please contact Disability Services at 614-292-3307 or slds@osu.edu. Further information: http://www.ods.osu.edu/

References

[1] D. Abramovich, L. Caporaso and S. Payne, *The tropicalization of the moduli space of curves*, Ann. Sci. Éc. Norm. Supér. **48**, no. 4 (2015), 765–809.

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[14] D. Maclagan and B. Sturmfels, *Introduction to Tropical Geometry.* Graduate Studies in Mathematics **161** (2015), Amer. Math. Soc., vii+359.

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