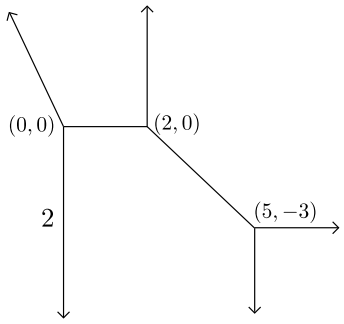
Reading course in Tropical Geometry – Problem set 2
Minkowski sums, plane interpolation, tropical hypersurfaces

Problem 1. Find a polynomial $f \in \mathbb{C}\{\{t\}\}[x,y]$ giving rise to the tropical plane curve below. Unless otherwise indicated, the multiplicity of an edge is assumed to be 1. The upper left ray has direction $(-1,2)$.



Problem 2. [Exercise 2.7.10 in [MS]] Pick 2 triangles P and Q that lie in non-parallel planes in $\mathbb{R}^3$.

(i) Draw their Minkowski sum $P+Q$ and its normal fan.

(ii) Write down the f-vector of $P+Q$ (i.e., describe how many faces of each dimension does $P+Q$ have).

(iii) Verify that the normal fan of $P+Q$ is the *common refinement* of the normal fans of P and Q.

Problem 3. Draw the *tropical hypersurface* associated to each of the following Laurent polynomials over the field $\mathbb{C}\{\{t\}\}$.

(i) $f_1(x,y) = t^3\,y^3 + y^2 - xy^2 - y - t^{-1}\,xy + x^2y + t^2 + x + x^2 + t^2\,x^3$;

(ii) $f_2(x,y) = xy + 5\,xy^2 - xy^3 + t\,x^2y + 3t^2\,x^2y^2 - 7t^2x^3y$;

(iii) $f_3(x,y) = t + xy + x^{-1}y + xy^{-1} + x^{-1}y^{-1}$;

(iv) $f_4(x,y,z) = 1 + 2x + 3y + 4z$;

(v) $f_5(x,y,z) = t\,x + y + z$.

Repeat the calculation for $f_4(x,y,z)$ over $\overline{\mathbb{Q}}_2$ and $\overline{\mathbb{Q}}_3$.
(*Useful Hint:* You might want to investigate how to do some of the previous examples using the software `Gfan` or the package `tropical.lib` in `Singular`.)