Jan 8

Chapter 1 of book

- solving simultaneous systems of linear equations
- meaning of linear
- Matrices
- Gaussian elimination & row echelon form
- Matrix Multiplication

Chapter 2:

- Vectors in $\mathbb{R}^2$ & $\mathbb{R}^3$
- Dot product

Chapter 3

- The vector space $\mathbb{R}^n$, subspaces
- Span of set of vectors
- linear independence, basis dimension
- Linear transformations = Matrix Multiplication

Vectors in plane or space

Algebraic vector in plane
= ordered pair of real numbers $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

in space = ordered triple

Addition: $v + w$

Scalar Multiplication: $c \in \mathbb{R}$

cv.

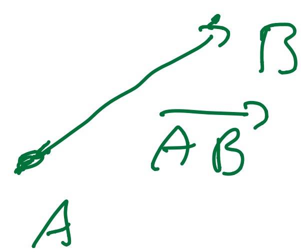
length:

Geometric vectors:
difference between 2 pts

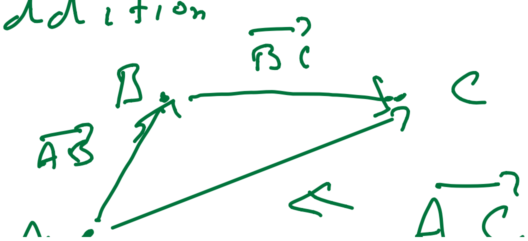
$A = (a_1, a_2)$

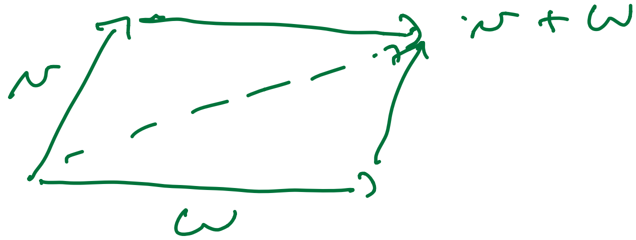
$B = (b_1, b_2)$

$\overrightarrow{AB} = \begin{bmatrix} b_1 - a_1 \\ b_2 - a_2 \end{bmatrix}$

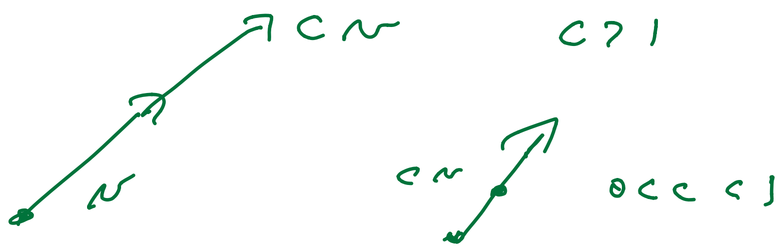


Geometric Interpretation of Addition





Scalar Mult c



Basic Properties of Addition & Scalar mult.

- $(u+v)+w = u+(v+w)$
- $u+v = v+u$
- $\exists$ a vector $\vec{0}$ $\left(= \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right)$ s.t. $v + 0 = v$
- For any v, $\exists$ vector $-v$ s.t. $v + (-v) = 0$

Scalar Mult. $r, s \in \mathbb{R}$

- $r(s v) = (rs) v$
- $r(v + w) = rv + rw$

Jan 10

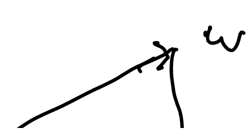
Vectors in $\mathbb{R}^2$ & $\mathbb{R}^3$

- linear combination
  (2 vectors in $\mathbb{R}^3$)
- Standard basis for $\mathbb{R}^2$, $\mathbb{R}^3$
- Other basis for $\mathbb{R}^2$
  say $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

  Write $\begin{bmatrix} 5 \\ 8 \end{bmatrix}$ as linear combination of these 2 vectors

  i.e solve linear system.
- linear independence
- Dot Product. $\vec{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$

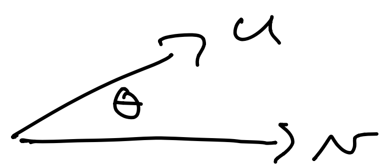
  $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ $\quad \vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2$
- $\text{proj}_{\vec{q}}(\vec{w}) = \vec{v}$



$$\operatorname{proj}_q(w) = \frac{w \cdot q}{\|q\|^2}\, q$$

$$= \frac{(\cos\theta)\|w\|\,\|q\|}{\|q\|}\,\frac{q}{\|q\|}$$

· $\vec{u} \cdot \vec{v} = |u|\,|v| \cos\theta$



· What does $\vec{u} \cdot \vec{v} = 0$ mean?

Def

A a <u>linear</u> <u>combination</u> of a set of vectors say in $\mathbb{R}^3$, $\{\vec{V}_1, \dots \vec{V}_n\}$ is combination

$$c_1 \vec{V}_1 + \cdot\ c_n \vec{V}_n$$

where $c_i \in \mathbb{R}$

Set of all linear combination $\{r\vec{v}_1 + s\vec{v}_2\}$ (v_1, v_2) is a plane passing through, called <u>span</u> of $\{\vec{v}_1, \vec{v}_2\}$

<u>Def</u> $\{v_1, \dots v_k\}$ is <u>basis</u> for $\mathbb{R}^n$ mean

- $\text{Span}\{v_1, \dots v_k\} = \mathbb{R}^n$
- $\{v_1, \dots v_k\}$ is lin independent

Ex in $\mathbb{R}^2$

$\{\vec{v}, \vec{w}\}$ is basis means

- any $\vec{u}$ is a linear combin.
- $r\vec{v} + s\vec{w} = \vec{0}$ has only trivial solut $r=0, s=0$

Jan 12

Assignment 2 (due Jan 24)

2.4: 2, 4, 10, 22
3.1 : 8, 15, 16, 19
3.2 : 1, 2, 3, 4, 10, 27, 28
3.7 : 4, 7
1.1 : 2, 5, 6, 8

Dot product of 2 vectors in $\mathbb{R}^n$, say $n=2$ or 3

$$\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \qquad \vec{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

$$\vec{v} \cdot \vec{w} = v_1 w_1 + v_2 w_2 + v_3 w_3$$

Basic Properties: $\vec{v} \cdot \vec{w}$ is

symmetric, bilinear form:

- $u \cdot (v+w) = u \cdot v + u \cdot w$

  $(v+w) \cdot u = v \cdot u + w \cdot u$

- $c \in \mathbb{R}$ $\quad u \cdot (cv) = c\,(u \cdot v)$

  $(cu) \cdot v = c\,(u \cdot v)$

- $u \cdot v = v \cdot u$ (symmetric)

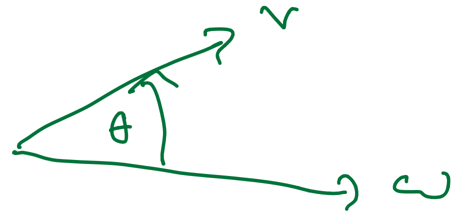
Also, $\vec{v} \cdot \vec{v} = v_1^2 + v_2^2 + v_3^2 \geq 0$

$= |\vec{v}|^2 \qquad (= \|\vec{v}\|^2)$

Key Formula: $\vec{v} \cdot \vec{w} = |\vec{v}|\,|\vec{w}| \cos\theta$

Def'n $\vec{v}, \vec{w}$ are <u>orthogonal</u>

if $\vec{v} \cdot \vec{w} = 0$



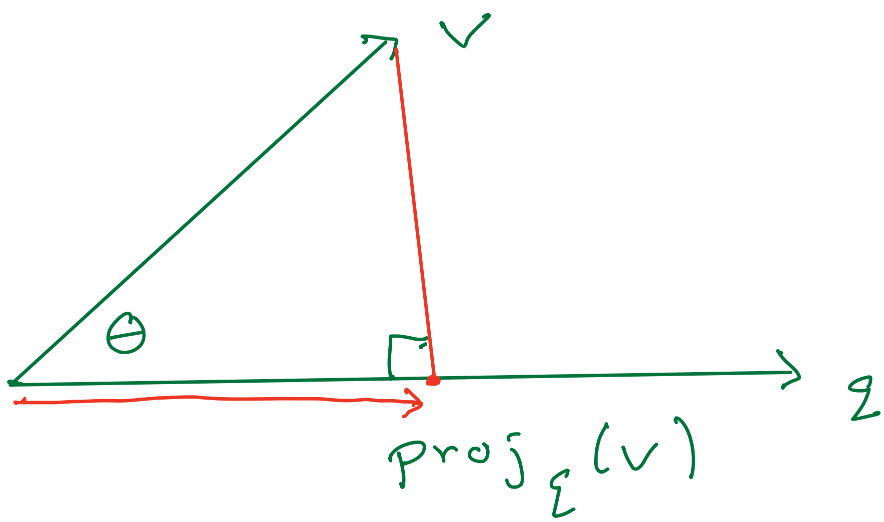
<u>Orthogonal projection</u>

v and q are vectors

Define $\operatorname{proj}_q(v)$ the

orthogonal projection of v

in direction q



$X = \mathrm{proj}_q (v)$

$|X| = |v| \, |\cos\theta|$

X should be parallel to q

$$X = \frac{(q \cdot v)}{(q \cdot q)} \vec{q}$$

$$= \frac{|q||w|\cos\theta}{|q|^2}$$

Ex $q = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ $v = \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}$

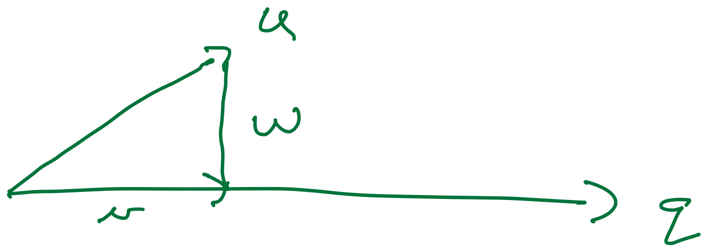
Then $X = \mathrm{proj}_q (v) = \frac{v \cdot q}{q \cdot q} q$

$$= \left(\frac{-4}{5}\right) \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} -4/5 \\ 0 \\ -8/5 \end{bmatrix}$$

Problem: Given vector u write it as sum of 2 vectors $u = v + w$ where

$v \cdot w = 0$

<u>Ans</u>



$v = \text{proj}_q(u)$

$w = u - v$

<u>Check</u> $v \cdot w = 0$

$v \cdot (u - v) = v \cdot u - v \cdot v \stackrel{?}{=} 0$