Linear Transformation $T: V \to W$

T is one-to-one $(N(T) = 0)$

T is onto $(R(T) = W)$

T is invertible (T is 1-1 and onto)

$B = \{\vec{v}_1, \dots \vec{v}_n\}$ = basis for V

- T one-to-one means $\{T(v_1), \dots T(v_n)\}$ is lin. ind.

- T is onto means $Sp\{T(v_1), \dots, T(v_n)\} = W$

- T is invertible means $\{T(v_1) \dots T(v_n)\}$ is basis for W

"<u>Coordinates</u>" (with respect to a basis)

$$B = \{ v_1, \ldots, v_n \}$$

$$v = x_1 \vec{v}_1 + \ldots x_n \vec{v}_n$$

$$[v]_B = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

Another way to say

$F : V \to \mathbb{R}^n$ by

$$F(v_i) = e_i$$

Then

$$F(\vec{v}) = [v]_B$$

<u>Ex</u> $\mathbb{P}_2$ $B = \{1, x, x^2\}$

$C = \{1, x-1, (x-1)^2\}$

<u>Ex</u> $T : \mathbb{R}^n \to \mathbb{R}^m$

$$T(e_i) = A_i$$

Then $T[\vec{x}] = A[\vec{x}]$

Matrix Rep of linear transformation

Feb 28

Linear Transformation $T: \mathbb{R}^n \to \mathbb{R}^m$

Its Matrix

Rotation MATRIX

Matrix of Projection

Linear Transformation between 2 vector spaces $T: V \to W$

Choose a basis $B = \{\vec{v}_1, \dots \vec{v}_n\}$ for V & $C = \{\vec{w}_1, \dots \vec{w}_n\}$ for W

$$\begin{array}{ccc} V & \xrightarrow{T} & W \\ \downarrow \cong & & \downarrow \cong \\ \mathbb{R}^n & \xrightarrow[F]{} & \mathbb{R}^m \end{array}$$

$$F([v]_B) \to [T(v)]_C$$

Ex $T: \mathbb{P}_2 \to \mathbb{P}_2 \quad T(p) = p'$

$T: \mathbb{R}^n \to \mathbb{R}^m$ is

determined by what

it does on a basis

$\{e_i\}$ standard basis for $\mathbb{R}^n$

$T(e_i)$ write in terms of standard basis for $\mathbb{R}^m$

$a_{1i}\hat{e}_1 + \cdots + a_{mi}\hat{e}_m$ $\hat{e}_j$

$T(e_i) = A_i$ = column vector in $\mathbb{R}^m$

$T(x_1e_1 + \cdots + x_ne_n) = x_1A_1 + \cdots + x_nA_n$

$\mathbb{R}^n$ $\qquad$ $= A\vec{x}$

$A = [A_1 \; \cdots \; A_n]$

Prob $T: \mathbb{R}^3 \to \mathbb{R}^3$

Multiply any v by 4.

Correspond to some 3x3 matrix

$$\begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \longrightarrow 4e_1 \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$$

$$e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}$$

March 2

- Meaning of dimension

- Vector space of matrices or polynomials

Ex. 3x3

- Linear Transformations
  - Choosing Bases & representing by matrix
  - Ex: $T: \mathbb{R}^n \to \mathbb{R}^m$
- Composition of Linear Transformations = matrix mult.
  - associativity of matrix mult
  - Invertible linear transformation
- Representing matrix mult by matrix (say 2x2)
- Projection onto line

$V = \{3 \times 3 \text{ matrices}\}$

9 dim

Basis

$$E_{11} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad E_{12} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad E_{13} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\shortparallel$$

$$E_{33}$$

$V = 2 \times 2$ symmetric

$$= \left\{ \begin{pmatrix} a & c \\ c & b \end{pmatrix} \right\}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = E_{12} + E_{21}$$

Matrix of linear transform

$$T: \mathbb{R}^n \longrightarrow \mathbb{R}^m$$

$\{e_1 \ldots e_n\}$ = standard

$$e_i \rightarrow \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

T corresponds

$$T(e_i) = \text{column vector } A_i.$$

$$A = [A_1, \dots, A_n]$$

$$Ae_i = A_i$$

V basis $B = \{v_1, \dots, v_n\}$

W basis $C = \{w_1, \dots, w_n\}$

$$\begin{tikzcd} V \arrow[r] \arrow[d, phantom, "\cong" rotate=90] & W \arrow[d, phantom, "\cong" rotate=90] \\ \mathbb{R}^n \arrow[r] & \mathbb{R}^m \end{tikzcd}$$

$T: V \to W$ linear transf

$T(v_i)$ = linear comb. of w_j

$= \cancel{a_{1i} w_1 + \dots a_{ni} w_n}$

$$[T(v_i)]_C = Q_i$$

$Q = [Q_1, \dots, Q_n]$ is matrix

corresponding to T

$V = 2 \times 2$ matrices

$B = \{E_{11}, E_{12}, E_{21}, E_{22}\}$

$T : V \to V$

$$T\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$\underset{A}{\parallel}$$

$$[A]_B = \begin{bmatrix} a_{11} \\ a_{12} \\ a_{21} \\ a_{22} \end{bmatrix}$$

$$T \longleftrightarrow \begin{array}{c} \begin{array}{cccc} E_{11} & E_{12} & & \end{array} \\ \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \\ 3 & 0 & 4 & 0 \\ 0 & 3 & 0 & 4 \end{pmatrix} \end{array}$$