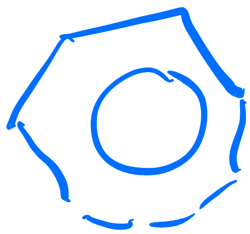


$\pi_1(X_{g'}) = \text{Free gp}$

$$\pi_1(M_g) = \pi_1(X_{g'}) \underset{\mathbb{Z}}{*} \pi_1(X_{h'})$$

$\mathbb{Z} = \pi_1(\partial\, \text{circle})$ "G"

<u>Thm</u> M_g is a $K(G,1)$

i.e. $\tilde{M}_g$ is contractible

(if $g \geq 1$)

Oct. 18

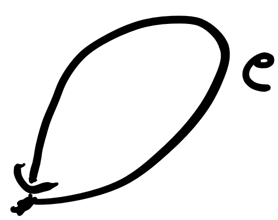
- Def'n of graph of groups
- Graph of spaces or graph of $K(G,1)$'s
- Mapping cylinders over half edges.

Thm. If all edge homomorphisms φ_e are injective, then $K\Gamma$ is a $K(G,1)$ and the inclusions $K(G_v,1) \hookrightarrow K\Gamma$ are injective on π_1.

Ex 1

$G = G_v *_{G_e} G_w$ = amalgam free product

Ex 2



$\varphi_0, \varphi_1 : G_e \to G_v$

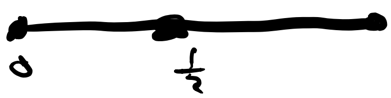
HNN extension

$G_v *_{G_e}$

$\pi_1(X) \xrightarrow{\varphi} K(G,1)$ $\exists !$

$f: X \to K(G,1)$

Gluing together $K(G,1)$'s



for each vertex

$X_v = K(G_v, 1)$ $\quad X_e = K(G_e, 1)$

e gives $f: X_e \longrightarrow X_v$

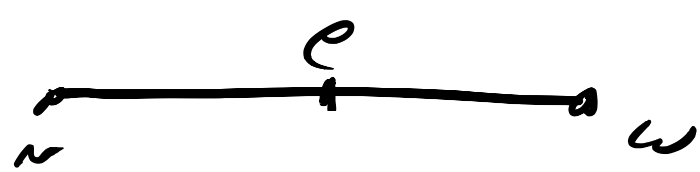
(~~½e~~) cover ½ edge

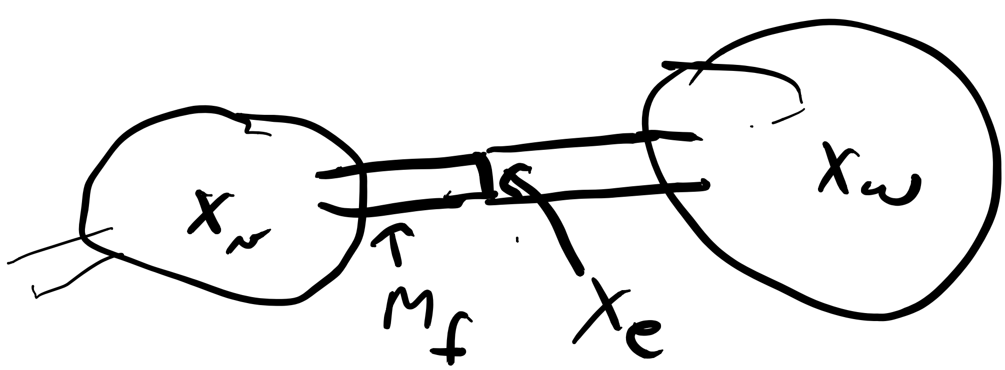
M_f = mapping cylinder

$$= \left(X_e \times [0, \tfrac{1}{2}]\right) / \sim$$

$$X_e \times 0 \xrightarrow{f} X_v$$

Glue together X_v & ~~X_e~~ M_f



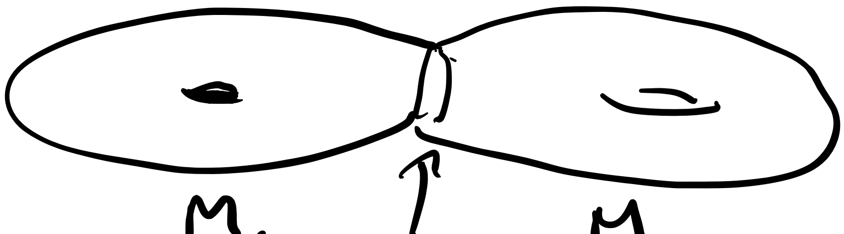


$= K_\Gamma$

$K_\Gamma = \bigcup$ mapping cylinder

$G_\Gamma = \pi_1(K_\Gamma)$

Standard Example



M_0 M_1

H

G_0 G_1 $H = \mathbb{Z}$

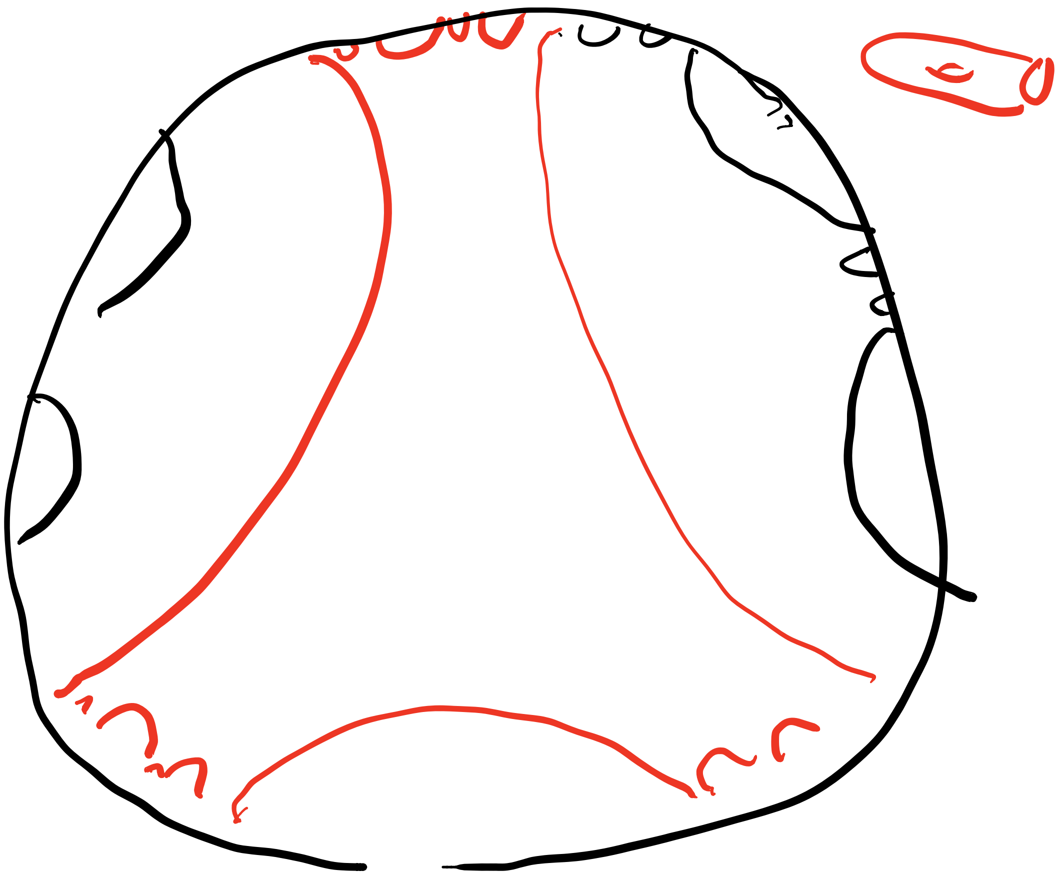
$\pi_1($  $)$ = free gp

γ

def retracts onto $\vee S^1$

~~Thm~~ ~~S~~ Surface $\sim M_0 \cup_{\partial M} M_1$

Thm $\Rightarrow$ M = Surface $\widetilde{M}$ is contractible



A, B are $K(G,1)$'s

$f: A \to B$ is injection on π_1

$M_f \overset{\text{def retract}}{\sim} B$

$\tilde{M}_f$ = univ. cover = contractible

$A \subset M_f$ $\quad \tilde{A}$ component of $p^{-1}(A) \subset \tilde{M}_f$, it is simply connected $\therefore$ contractible

~~Many com~~

Mapping cylinder of $\tilde{A} \to \hat{B}$

<u>Idea</u> of <u>Pf</u>

$$\underset{G_v}{\bullet}\overset{G_e}{\rule{3cm}{0.4pt}}\underset{G_w}{\bullet}$$

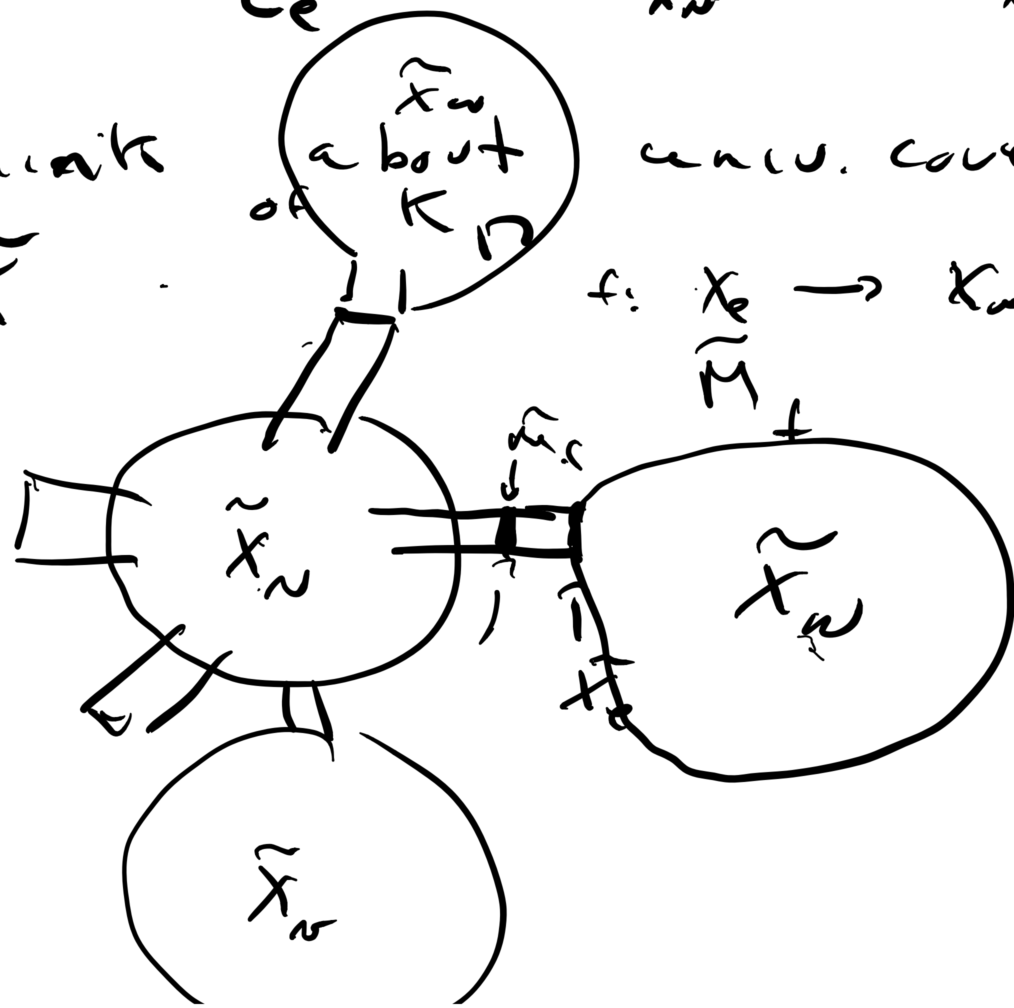
$X_v \qquad X_w$

$G = G_v *_{G_e} G_w$

Think about univ. cover of K

$\tilde{K}$.

$f: X_e \to X_w$



Hatcher Proof: Construct

spaces $K_0 \subset K_1 \subset K_2 \cdots$

s.t. $\tilde{K} = \bigcup K_n$ is

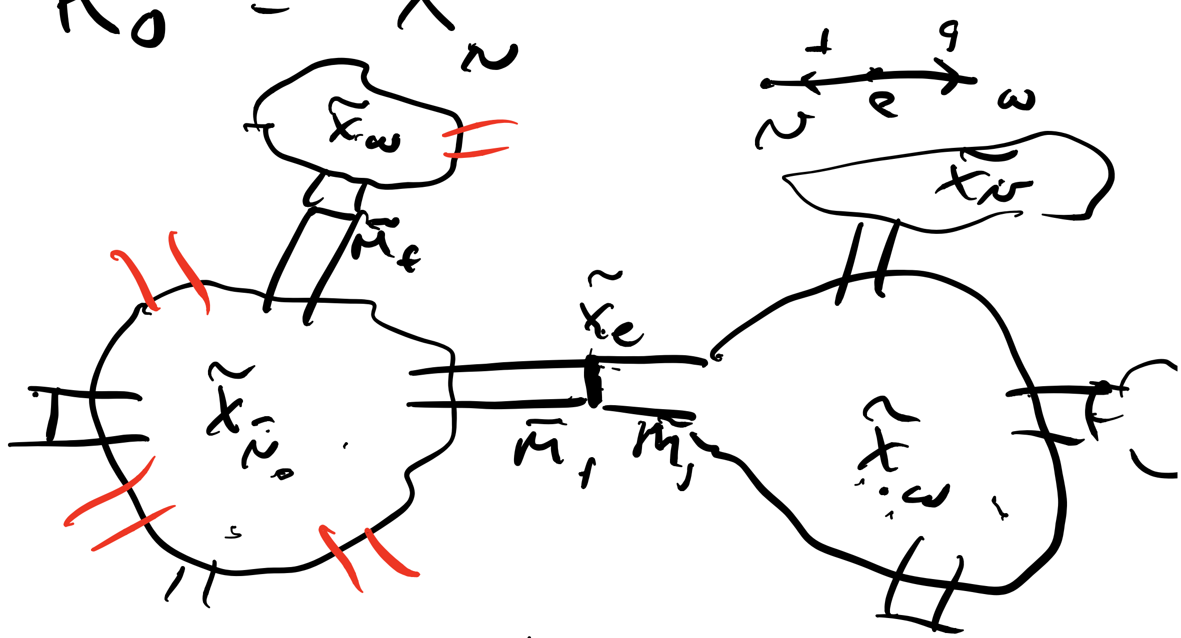
a cover of $K\Gamma$

Replace X_v by $X_v \cup M_f$

$\cup M_f$ for each $e \ni v$.

Pick one M_f, i.e. one $\tilde{X}_v$

$K_0 = \tilde{X}_v$



$K_1 = K_0 \cup \bigcup$ indicated $\tilde{X}_\omega$

Repeat to get K_2

$\tilde{K}$ = union

$p : \tilde{K}_n \longrightarrow K\Gamma$

$p : \tilde{K} \longrightarrow K\Gamma$

is a covering

$\tilde{K}$ is simply connected

(in fact, contractible

$\tilde{K}_{n+1} = \tilde{K}_n \cup$ Contractible

$\tilde{K}_{n+1} = \tilde{K}_n \underset{\substack{\uparrow \\ \text{contractible}}}{\cup}$ Contractible

van Kampen simply connected

... $\tilde{K}$ is contractible.

$$G_N \longrightarrow \pi_1(K\Gamma)$$

is injective ..... .

Connection to Groups acting on trees.

$G = G_\Gamma = \pi_1(K\Gamma)$

$G \curvearrowright \tilde{K}$ preserving decomp

T = tree with one vertex for each $\tilde{X}_v$ (component)

edge for each $\mathbb{I}$

$G \curvearrowright T$ Serre's <u>Trees</u>

T is infinite valence at vertex

~~G_{v}~~

Construct G

$\mathrm{vert}(T) = \coprod G/G_v$

$$\mathrm{Edges}(T) = \bigsqcup G/G_e$$

$$\Gamma = T/G$$