Index of notations

Set theory

$\emptyset$ is the empty set.
$x \in X$ denotes that x is an element of the set X.
$X \subseteq Y$ means that X is a subset of Y.
$\bar{X}$ denotes the closure of X (in Y).
$\{x \mid A(x)\}$ is the set of objects x satisfying a condition $A(x)$.
$X \times Y$ is the set of pairs $\{(x, y) \mid x \in X, y \in Y\}$.
$\Delta(X)$ is the diagonal subset $\Delta(X) := \{(x, y) \in X \times X \mid x = y\}$ of $X \times X$.
$f : X \to Z$ means that f is a map from X to Z.
$\operatorname{Im} f$ denotes the image of f: $\{f(x) \mid x \in X\}$.
$|$ denotes restriction, for example, for $Y \subseteq X \xrightarrow{f} Z$, $f|Y$ is the restriction of f to Y.
$\circ$ denotes composite: the composite $g \circ f$ is given by $g \circ f(x) := g(f(x))$.
$\mathbb{R}$ is the set of real numbers,
$\mathbb{R}^n$ is the vector space of n-tuples $x = (x_1, \dots, x_n)$ with each $x_k \in \mathbb{R}$,
$\|x\| := \sqrt{(x_1^2 + \cdots + x_n^2)}$.
$\mathbb{R}^n_+$ is the subset with $x_1 \geq 0$, $\mathbb{R}^n_{++}$ the subset with $x_1 \geq 0, x_2 \geq 0$.
$[a, b]$ is the closed interval $\{x \in \mathbb{R} \mid a \leq x \leq b\}$;
$[a, b)$ is the half-open interval $a \leq x < b$ (allowing $b = \infty$); similarly $(a, b]$.
$D^n_x(r)$ is the closed disc $\{y \in \mathbb{R}^n \mid \|x - y\| \leq r\}$,
$S^{n-1}_x(r)$ the sphere $\{y \in \mathbb{R}^n \mid \|x - y\| = r\}$,
$\mathring{D}^n_x(r)$ the open disc $\{y \in \mathbb{R}^n \mid \|x - y\| < r\}$,
$D^n_+(r) := D^n_x(r) \cap \mathbb{R}^n_+$ is the closed half-disc
$\mathring{D}^n_+(r) := \mathring{D}^n_x(r) \cap \mathbb{R}^n_+$ the open half-disc.

If x is omitted, the centre is the origin; if r is omitted, the radius is $r = 1$.
$D^k(a, b) := \{x \in \mathbb{R}^k \mid a \leq \|x\| \leq b\}$.

Thus $I := [0, 1] = D^1_+$ and $\mathbb{R}^+ := \mathbb{R}^1_+ = [0, \infty)$.
$u : \mathbb{R}^n \setminus \{0\} \to S^{n-1}$ is defined by $u(x) := x/\|x\|$. (§4.2).

Groups, fields, etc.

$\mathbb{Z}$ is the ring of integers.
$\mathbb{Z}_n$ is the additive group of integers modulo the natural number n.
$\mathbb{R}$ is the field of real numbers.
$\mathbb{Q}$ is the field of rational numbers.
$\mathbb{C}$ is the field of complex numbers.
$\sigma(q)$ is the signature of a quadratic form q defined over $\mathbb{R}$,
$\mathrm{Arf}(\mu)$ is the Arf invariant of a quadratic form μ defined over $\mathbb{Z}_2$.
$\mathfrak{G}(\mu)$ is the Gauss sum of the quadratic form μ on a finite group.
$P(A_*; F)(t) := \sum_0^\infty \dim_F(A_n)t^n$ is the Poincaré series of A_*.
$|G|$ is the order of the finite group G.
$\mathrm{Tors}(A)$ is the torsion subgroup of the abelian group A.
$A \oplus B$ is the direct sum of A and B;
$A \otimes B$ is the tensor product of A and B.
$G^\vee$ is the dual group to G.
$\mathrm{Ker}(\phi)$ is the kernel of the group homomorphism $\phi : A \to B$;
$\mathrm{Coker}(\phi)$ is the cokernel of $\phi : A \to B$.
G/H is the quotient (space) of (right cosets) of a group G by a subgroup H. If H is a normal subgroup of G, this is the quotient group.

$\mathbf{GL}_m(K)$ is the group of nonsingular $(m \times m)$ matrices over the field K.
$\mathbf{SL}_m(K)$ is the subgroup of matrices of determinant 1.
$\mathbf{GL}^+_m(\mathbb{R}) \subset \mathbf{GL}_m(\mathbb{R})$ is the subgroup of matrices with positive determinant.
$\mathbf{O}_m \subset \mathbf{GL}_m(\mathbb{R})$ is the orthogonal group, $\{A \in \mathbf{GL}_m(\mathbb{R}) \mid AA^t = I\}$.
$\mathbf{U}_m \subset \mathbf{GL}_m(\mathbb{C})$ is the unitary group, $\{A \in \mathbf{GL}_m(\mathbb{C}) \mid A\overline{A}^t = I\}$.
$\mathbf{SO}_m := \mathbf{O}_m \cap \mathbf{SL}_m(\mathbb{R})$.
$\mathbf{SU}_m := \mathbf{U}_m \cap \mathbf{SL}_m(\mathbb{C})$.
G_n is the monoid of maps of S^{n-1} to itself of degree ± 1.
$F_n \subset G_{n+1}$ is the set of base-point preserving maps $S^n \to S^n$.
Top_n is defined in §8.9.
SG_n, SF_n, $STop_n$ are the corresponding subsets of orientation-preserving maps.

For each of the above groups and monoids C_n,
$B(C_n)$ is the classifying space of C_n;
$B(G_n)$ is the classifying space for spherical fibrations with fibre S^{n-1};
C is the union of the C_n; and
$B(C)$ is the inductive limit of the sequence $B(C_n)$.

Manifolds, etc.

$Bp(x)$ is the bump function (§1.1).
$T_P M$ is the tangent space at $P \in M$ to the smooth manifold M;
$T_P^\vee M$ is the dual vector space.
$\mathbb{T}(M)$ is the tangent vector bundle of M; the dual is $\mathbb{T}^\vee(M)$.
$\mathbb{T}^0(M)$ is the zero cross-section.
$\mathbb{N}(M/V)$ is the normal bundle of the smooth submanifold $V \subset M$.
∂M is the boundary of M: for example, $\partial D^n_x(r) = S^{n-1}_x(r)$.
$\angle M$ is the corner of M.
$\mathring{M} := M \setminus \partial M$ is the interior of M.
$\partial_- W$, $\partial_+ W$ and $\partial_c W$ are the lower, upper, and middle parts of the boundary ∂W of a cobordism W.
$D(M)$ is the double of M.
$M_1 \# M_2$ is the connected sum of manifolds M_1 and M_2.
$M_1 + M_2$ is the boundary sum of M_1 and M_2 (§2.7).
$P^m(\mathbb{R}) = P(\mathbb{R}^{m+1})$ is the set of lines through the origin in $\mathbb{R}^{m+1}$;
$P^m(\mathbb{C}) = P(\mathbb{C}^{m+1})$ the set of lines in $\mathbb{C}^{m+1}$.
$P^\infty(\mathbb{R}) := \bigcup_{n\in\mathbb{N}} P^n(\mathbb{R})$; $P^\infty(\mathbb{C}) := \bigcup_{n\in\mathbb{N}} P^n(\mathbb{C})$.
$Gr_{m,k}$ is the Grassmann manifold of k-dimensional subspaces of $\mathbb{R}^m$.
$V_{m,k} \cong O_m/O_k$ is the Stiefel manifold of isometric embeddings $\mathbb{R}^k \to \mathbb{R}^m$.
$V'_{m,k} \cong GL_m(\mathbb{R})/GL_k(\mathbb{R})$ is the set of linear embeddings $\mathbb{R}^k \to \mathbb{R}^m$.
$J^k(V, M)$ is the space of k-jets of maps $V \to M$;
$j^k f : V \to J^k(V, M)$ is the k-jet of the map $f : V \to M$;
$V^{(r)}$ is the subset of V^r consisting of r-tuples of *distinct* points of V.
${}_rJ^k(V, M)$ is the subset of $(J^k(V, M))^r$ lying over $V^{(r)}$.
${}_rj^k f : V^{(r)} \to {}_rJ^k(V, M)$ is the multijet of $f : V \to M$. §4.4.
$C^r(V, M)$ is the set of C^r maps $V \to M$ ($0 \le r \le \infty$);
$C^r_{pr}(V, M)$ is the set of proper C^r maps.
$\mathrm{Imm}(V, M)$ is the set of (smooth) immersions $V \to M$;
$\mathrm{Emb}(V, M)$ is the set of (smooth) embeddings $V \to M$;
$\mathrm{Diff}(M)$ is the set of diffeomorphisms of M.
Σ^i, $\Sigma^i(V, M) \subset J^1(V, M)$, $\Sigma^i f$ are Thom-Boardman sets: see §4.5.
$I(\phi)$ is the number of double points of an immersion $\phi : V^k \to M^{2k}$.

Cobordism theory

For ξ an orthogonal bundle (or spherical fibration), we write
A_ξ for the associated disc bundle,
S_ξ for the sphere bundle,

$T(\xi) = A_\xi / S_\xi$ for the Thom space,
B_ξ for the base.
For ξ the universal bundle over $B(C_n)$, these become $A(C_n)$, $S(C_n)$, $T(C_n)$.
$\Omega_m(X, \nu)$ is the set of normal cobordism classes of maps of degree 1 to X.
$P_m := \Omega_m(D^m, \varepsilon)$.
$\mathrm{Kerv}(\phi, \nu, T)$ is the Kervaire invariant of a normal cobordism class.
L_m is $\mathbb{Z}$, 0, $\mathbb{Z}_2$ or 0 according as $m \equiv 0, 1, 2$ or 3 (mod 4).
Ω_m^G is the cobordism group of m-manifolds with (weak) G-structure.
Ω_m^{fr} is the framed cobordism group.
Θ_m^k is the group of homotopy spheres $\Sigma^m \subset S^{m+k}$,
$F\Theta_m^k$ is the group of framed homotopy spheres $\Sigma^m \subset S^{m+k}$,
Σ_m^k is the group of embeddings $S^m \subset S^{m+k}$.
B_k is the kth Bernoulli number.

Homology theory

$H_r(X, Y; A)$ is the rth homology group of (X, Y) with coefficients in A.
If A is omitted, it is taken as $\mathbb{Z}$.
$\tilde{H}^k(X, Y)$ is the reduced cohomology group.
$K_k(M) := \mathrm{Ker}(\phi_* : H_k(M) \to H_k(X))$ for $\phi : M \to X$ a normal map.
$[M]$ is the fundamental homology class of the manifold M.
$\beta_p : H^k(X; \mathbb{Z}_p) \to H^{k+1}(X; \mathbb{Z}_p)$ is the Bockstein homomorphism.
$\mathcal{S}_p$ is the mod p Steenrod algebra,
χ its canonical anti-automorphism,
$\overline{\mathcal{S}}_p := \mathcal{S}_p / \langle \beta_p \rangle$.
$K(\pi, n)$ is the Eilenberg–MacLane space.
$J_k : \pi_k(SO) \to \pi_k^S$ is the stable J homomorphism.
$w_k(\xi), v_k(\xi) \in H^k(X : \mathbb{Z}_2)$ are the Stiefel–Whitney, Wu classes of a bundle ξ.
If $\xi \oplus \eta$ is trivial, $\overline{w}_k(\xi) = w_k(\eta)$.
$c_k \in H^{2k}(X; \mathbb{Z})$ is the kth Chern class,
$p_{4k} \in H^{4k}(X; \mathbb{Z})$ are the Pontrjagin classes.

Homotopy theory

$*$ is the base point.
X^+ is the disjoint union of X and $*$.
$X \wedge Y$ is the smash product of X and Y.
$X * Y$ is the joint of X and Y.

$[X : Y]$ is the set of (based) homotopy classes of maps $X \to Y$.
$\pi_r(X, Y)$ is the rth homotopy group of (X, Y).
$K^{(n)}$ is the n-skeleton of K.
$X^{\langle k \rangle}$ is a $(k-1)$-connected cover of X.
$SX := S^1 \wedge X$ is the suspension of X.
ΩX is the loop space of X.
$\{X : Y\} = lim_{n\to\infty}[S^n X : S^n Y]$.
$\pi_r^S(X) := \{S^r : X\}$.
$\mathbb{S}$ is the sphere spectrum.
$\mathbb{K}(A, k)$ is the Eilenberg–MacLane spectrum.
$\mathbb{TG}$ is the classifying spectrum of the stable group G (in the sense of §8.2).
$\mathbb{BU}$ and $\mathbb{BO}$ are the Bott spectra, with connective versions $\mathbb{BU}\langle k \rangle$ and $\mathbb{BO}\langle k \rangle$.