CORRECTION TO
"THE TOPOLOGY AT INFINITY OF COXETER GROUPS AND BUILDINGS"

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The paper referred to in the title concerns the algebraic topology at infinity of geometric realizations of Coxeter groups and of buildings. For Coxeter groups, the arguments are correct; however, for buildings, they are not. Jan Dymara and Damian Osajda pointed out to us some serious problems with Section 5, where the results for buildings are given (see [4]). In particular, Lemmas 5.3 and 5.7 are wrong. This leads to gaps in the proofs of Theorems 5.8, 5.12 and 5.13. Nevertheless, we believe that the theorems in Section 5 remain true. (More precisely, Theorems 5.12 and 5.13 should hold as stated and a version of Theorem 5.8 should be true.)

The basic mistake in Section 5 is this. Suppose $C$ is (the set of chambers of) a building and $X \subset C$ is a subset which is starlike with respect to a base chamber $c_0$. Let $c$ be an extreme chamber in $X$ and put $\tilde{X} := C - X$. Implicit in both Lemmas 5.3 and 5.7 is the assumption that

\[ |\tilde{X}| \cap |c| = |c|^{I_1(X,c)}. \]

In other words, the intersection on the left is a certain union of mirrors of $|c|$ indexed by $I_1(X,c)$, where $I_1(X,c)$ denotes the set of $i \in I$ such that $\tilde{X}$ contains a chamber $i$-adjacent to $c$. In fact, the intersection in (0.1) need not be a union of mirrors. For example, it can be the union of $|c|^{I_1(X,c)}$ with a lower dimensional face. (This can be seen even in the case of thick, right-angled, spherical buildings of rank $\geq 2$.)

In the calculation of $H^*_c(|C|)$ in Theorem 5.8, one starts by ordering $C, c_0, c_1, \ldots$, so that $l(\delta(c_0, c_{k+1})) \geq l(\delta(c_0, c_k))$, where $\delta( , )$ is the W-valued distance on $C$ and $l( )$ is word length on $W$. If $X_m := \{c_0, c_1, \ldots, c_m\}$, then one wants formula (0.1) to hold with $X = X_m$ and $c = c_m$. There is considerable freedom in choosing the ordering of $C$ and not all choices work. To see this, suppose $R$ is a spherical residue in $C$ and $d_R \in R$ is its chamber closest to $c_0$. Let $L_R$ be the set of chambers in $R$ which are furthest from $d_R$. ($L_R$ is the set of chambers in $R$ opposite to $d_R$.) Since the elements of $L_R$ all have the same $W$-valued distance from $c_0$, when choosing the ordering of $C$, $L_R$ can be ordered arbitrarily. Most choices of orderings will not satisfy (0.1). In particular, if $L_R$ is not gallery-connected, no choice will work.

When $C$ is right-angled the situation can be remedied. For in this case, $L_R$ is a spherical building of the same type $(W_J, J)$ as $R$ (at least when $R$ is thick). Order the elements of $L_R$ using the $W_J$-distance on $L_R$. (It may be necessary to apply this step repeatedly.) The conclusion is that for right-angled buildings there is an ordering of $C$ satisfying (0.1). Hence, Theorems 5.8, 5.12 and 5.13 hold for right-angled buildings and Corollary 5.14 is true as stated. A different proof of Theorem 5.8 for right-angled buildings is given in [2]. It also uses the fact that $L_R$ is a building of type $(W_J, J)$. In the right-angled case, Theorems 5.12 and 5.13, as well as, Corollary 5.11 also follow from the results in [1].
References


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