I BS Bordifications

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VI Curve cx of spherical Artin group

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M = aspherical mfld

$\widetilde{M}$ = universal cover

$G = \pi_1(M)$

Def'n. A BS-bordification of $\widetilde{M}$ is a mfld with corners $\overline{M}$ with G-action s.t.

- $\overline{M}/G$ is compact
- $\widetilde{M} = \operatorname{int} \overline{M}$
- $\exists$ simplicial cx $\mathcal{C}$
  simplices of $\mathcal{C} \longleftrightarrow$ strata $\overline{M}$

k-simplex $\longleftrightarrow$ codim $k+1$ stratum

- Strata are contractible
- $\mathcal{C} \sim \bigvee(m\text{-spheres})$.

Consequence: G is a duality gp of dim $n-m-1$.

(Conversely, if G is duality gp & $\partial\overline{M}$ is 1-connected, then $\mathcal{C} \sim \bigvee S^m$.)

Pf. $\partial\overline{M} \sim \mathcal{C} \sim \bigvee S^m$. So

$$H^*(G;\mathbb{Z}G) = H^*_c(\overline{M})$$

$$= H_{n-*}(\overline{M}, \partial\overline{M})$$

$$= \tilde{H}_{n-*-1}(\partial\overline{M}) = \tilde{H}_{n-*-1}(\mathcal{C})$$

$\neq 0$ only for $* = n-m-1$. $\square$

II Examples

- <u>Arithmetic gps</u>, say $G = GL(n,\mathbb{Z})$

$\tilde{M} = X =$ symmetric space

$M = X/G$

$\mathcal{C} =$ spherical bldg for $GL(n,\mathbb{Q})$

$= (n-1)$-dim simpl. cx

$\mathrm{Vert}\,\mathcal{C} = \{$ maximal parabolic subgps $\}$

i.e conjugate of $\begin{bmatrix} * & * \\ 0 & * \end{bmatrix}$

Codim 1-stratum = $\begin{pmatrix}\text{symmetric} \\ \text{space}\end{pmatrix} \times \begin{pmatrix}\text{symmetric} \\ \text{space}\end{pmatrix} \times \cancel{\mathbb{R}^+}$

$\times$ Nilpotent

$C-$

- MCG (surface)

$\tilde{M}$ = Teichmuller Space

$\mathcal{C}$ = curve cx

Vert $\mathcal{C}$ = $\left\{ \begin{array}{l} \text{isotopy classes of} \\ \text{simple closed curves} \end{array} \right.$

codim 1 stratum = Teichmüller space for nodal surface $\times \mathbb{R}_+$

- <u>Pure Artin gps</u> & hyperplane arrangements

W = finite Coxeter gp $W \curvearrowright \mathbb{R}^n$

$\therefore$ on $\mathbb{C}^n$.

$\mathcal{A} = \left\{ \begin{array}{l} \text{reflecting hyperplanes} \\ \text{in } \mathbb{C}^n \end{array} \right\}$

$M = \mathbb{C}^n - \bigcup H$

$H \in \mathcal{A}$

$G = \pi_1(M) = PA$

$A = \pi_1(M/W)$

$\widetilde{M}$ = universal cover

__Goal:__ Describe $\overline{M}$ & G

III Blowing up complements of hyperplane arrangements

$\mathcal{A}$ = {hyperplanes} in $V = \mathbb{C}^n$

subspace = intersection of hyperplanes in $\mathcal{A}$

Q = intersection poset = {subspaces}

If $E \in Q$, $\mathcal{A}_E = \{H \in \mathcal{A} \mid E \leq H\}$

$\mathcal{A}^E = \{E \cap H \mid H \in \mathcal{A} - \mathcal{A}_E\}$

viewed as arrangement in V/E.

$$V_0 = V - \bigcup_{H \in \mathcal{A}} H$$

$$S(V/E)_0 = S(V/E) - \bigcup_{H \in \mathcal{A}_E} S(H/E)$$

Fast def'n of Blow-up of family of subspaces $\mathcal{E}$

$$\rho : V_0 \longrightarrow V \times \prod_{\mathcal{E}} S(V/E)$$

$$V_{\odot} = \overline{\rho(V_0)}$$. Mfld with corners

codim 1 stratum will have form

$$E_0 \times S(V/E)_0$$

IV Irreducible subspaces

<u>Def'n</u> $E \in Q$ is <u>reducible</u> if $\mathcal{A}_E$ splits as a direct sum

$$V/E = V/F_1 \oplus V/F_2$$

<u>Def'n</u> of <u>irreducible cx</u> $\mathcal{I}$

$$\text{Vert}\, \mathcal{I} = \{E \in Q \mid E \text{ is irreducible}\}$$

Two types of edges $\{E, F\}$

- <u>comparable</u> $E < F$ or $F < E$
- <u>commuting</u> : Means $E \cap F$ reducible

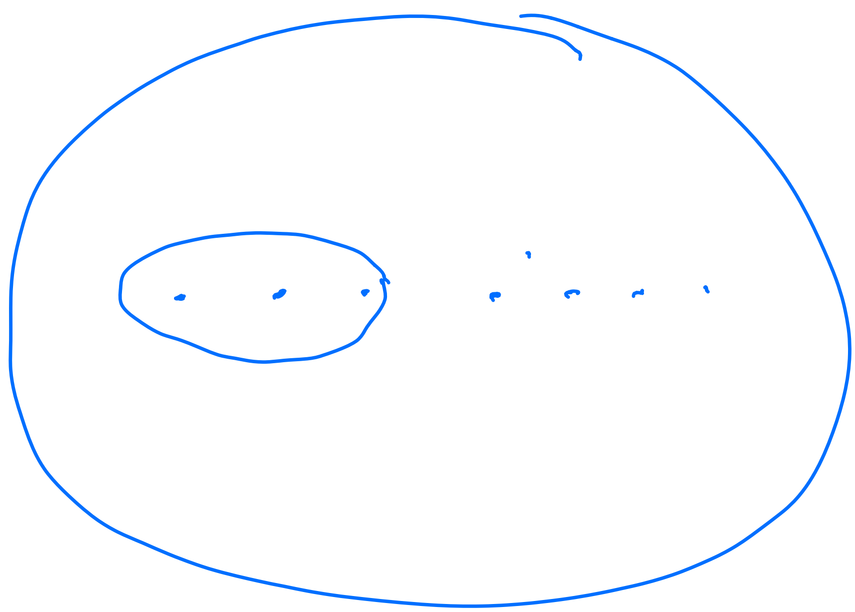
$$V/E\cap F = E/E\cap F \oplus F/E\cap F$$

$$= V/F \oplus V/E$$

$\mathcal{I}$ = associated flag cx
(This is homotopic to V(sphere))

V Braid gp

Relationship between curve cx for PB_n & irreducible cx I for braid arrangement



single loop = irreducible subspace

Pair of nested loops = comparable

unnested loop = reducible subsp.
= commuting pair

VI. Two definitions of the curve complex $\mathcal{C}$

First Definition:

$$G = PA = \pi_1(V_\odot)$$

For each simplex $\sigma \in \mathcal{I}$

$G_\sigma = \pi_1(\partial_\sigma V_\odot)$. If

$\sigma = E$ a vertex of $\mathcal{I}$, then

$$\partial_\sigma V_\odot = E_\odot \times S(V/E)_\odot$$

Then G_σ is a subgp of G

Then $\mathcal{C}$ is the corresponding

simplicial cx of left

cosets $\coprod_{\mathcal{I}} G/G_\sigma \,/\!\sim$

Thm 1) G is the development of a simple cx of gps over $\mathcal{I}$.

2) $G \sim \bigvee S^{n-2}$, $n = \dim V$

(Squier)

Cumplido, Gebhardt, Gonzalez-, Weiss

Second Definition: In terms of "parabolic subgps"

For each $E \in \mathrm{Vert}\,\mathcal{I}$, put

$$P_E = \pi_1(S(V/E)_{\mathbb{C}})$$

Any conjugate of P_E in G is an irreducible spherical parabolic subgp

These parabolic subgps are

Vert $(\mathcal{C})$. (Intersections of parabolics are parabolics $CG \subseteq W$)

$\{$irreducible parabolic subgps$\}$ = Vert $\mathcal{C}$

$\{$Centers of " "$\}$ = Vert $\mathcal{C}$

Fill in edges & simplices as before.

G acts on $\mathcal{C}$ by conjugation

Stabilizer of P_E is

$$N(P_E) \overset{?}{=} Z(\text{Center}(P_E))$$

$$= \pi_1(E_\odot) \times \pi_1(S(V/E_\odot))$$

$$= G_E$$

So we get the same

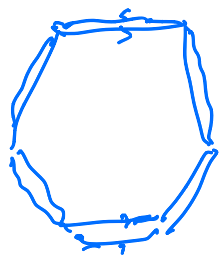
simpl. cx $\mathcal{C}$ as before.

$\boxed{\text{Cumplido Gebhardt Gonzales-Meneses}}$

& Weist.

Def^{n}s from $\widetilde{Sal}$ = univ cover of Sal

Sal = doubled zonotope

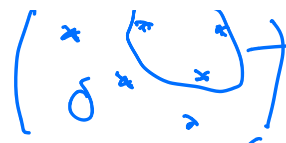


Parallel Class of faces $\longleftrightarrow$ subspa

Standard Parabolic = π_1(subcomplex corresponding to face)

Uri, Pierre-Emmanuel, Tadeusz, Piotr Nowak, Damain

$\begin{pmatrix} 1 & * \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} * & * \\ 0 & * \end{pmatrix}$



<u>Question</u> : Is G hyperbolic?