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MATH 6701, AUTUMN 2024
M-W-F 10:20 a.m., MP 1035
A DAY-BY-DAY LIST OF TOPICS

Included are references to the following texts, accessible through the course homepage at
`https://people.math.osu.edu/derdzinski.1/courses/6701/6701.html`

[**DG**]: *Differential Geometry*, [**DF**]: *Distributions and the Frobenius Theorem*,
[**PS**]: *Projective Spaces and Grassmannians*, [**LL**]: *Local Lie-group structures*

August 21: Topological spaces, open sets, neighborhoods, continuity of mappings, convergence of sequences. The Hausdorff, first-countability and second-countability axioms. Connectedness and pathwise connectedness of sets in topological spaces, their preservation under continuous images, and the fact that the latter implies the former (since the union of any nonempty family of connected sets with a nonempty intersection is necessarily connected, while intervals are connected subsets of $\mathbb{R}$). Connected components of a topological space (the equivalence classes of the relation in which two points are called equivalent if they both lie in a connected set). The observation that the connected component of a point x is the union of all connected sets containing x, as well as the largest connected set containing x. Simultaneous openness and closedness of the connected components when the space is locally connected (meaning that every neighborhood of a point x contains a connected neighborhood of x). Closedness and compactness of subsets in topological space, the former meaning openness of the complement [**DG**, p.2], the latter involving the open-covering test [**DG**, p.54]. Closedness in terms of convergent sequences when the first-countability axiom is assumed. Compactness of continuous images of compact sets [**DG**, p.9] and their finite unions. Some consequences of the first-countability axiom: closedness of compact sets and compactness of their closed subsets; the fact that a bijective continuous mapping from a compact topological space is a homeomorphism. A family of sets subordinate to another family. Equivalence of compactness to sequential compactness [**DG**, p.4] in spaces satisfying the second-countability axiom: the Heine-Borel theorem [**DG**, p.54]. Metric spaces. Charts and C^k compatibility, $k = 0, 1, \ldots, \infty, \omega$. Definition of an n-dimensional C^k atlas. Maximal atlases. The existence of a unique maximal atlas containing a given one [**DG**, p.1].

August 23: The topology resulting from an n-dimensional C^k atlas on the given set. Definition of a C^k (Hausdorff) manifold [**DG**, pp.1–2] including the second-countability axiom [**DG**, p.53]. Equivalence of the latter axiom to the existence of a countable subatlas of the maximal atlas. (A manifold is thus a set with a fixed maximal atlas satisfying the Hausdorff and countability axioms.) Geometric properties, including openness of sets [**DG**, p.4]. Examples of manifolds: zero-dimensional ones (that is, nonempty countable sets); open submanifolds [**DG**, p.5]; Cartesian products of manifolds [**DG**, p.4]; vector spaces [**DG**, pp.2]; affine spaces [DG, pp. 3 and 191-192]. Euclidean spheres centered at zero: $M = \{x \in V : \langle x, x\rangle = a^2\}$ in a Euclidean space V with the inner product $\langle\,,\rangle$, where stereographic projections serve as chart mappings [**DG**, p.3]. The formula $x \mapsto y = z + 2a^2(x - z)/(a^2 - \langle z, x\rangle)$ for the stereographic projection $M \smallsetminus \{z\} \to z^\perp - z$ from the pole $z \in M$, and $y \mapsto x = z + 2(a^2 - \langle z, x\rangle)(x - z)/\langle x - z, x - z\rangle$ for its inverse.