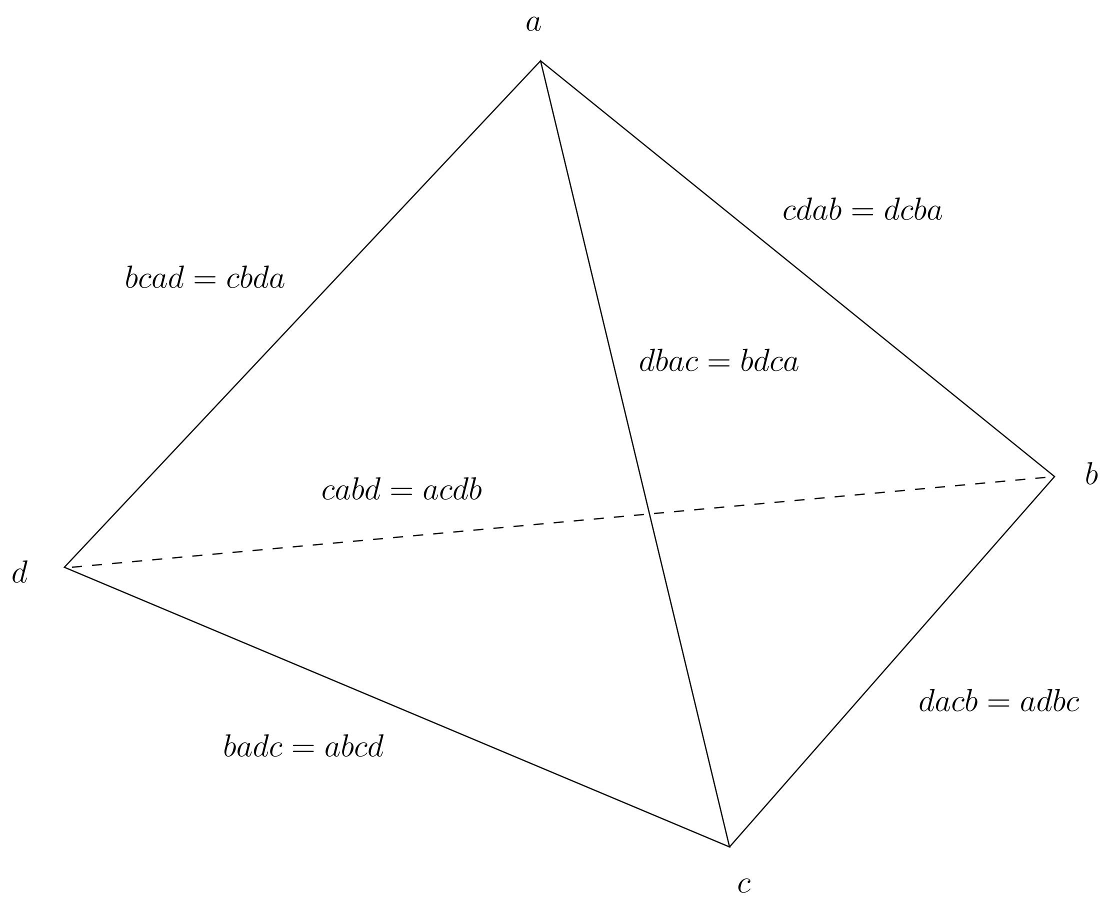

The fact that every algebraic curvature tensor R also has $R(u, u', v, v') = R(v, v', u, u')$ follows, with the ad hoc notation $abcd$ for $R(a, b, c, d)$, via the following steps: $-2abcd = -abcd - badc = (bcad + cabd) + (adbc + dbac) = (cbda + bdca) + (acdb + dacb) = -dcba - cdab = -2cdab$ (summary: place c, d at the end, use Bianchi, place a, b at the end, use Bianchi).

The tetrahedron version of Milnor's octahedron argument shows this by marking the four vertices of a tetrahedron with a, b, c, d, and distributing the twelve values such as $abcd$, corresponding to even permutations of a, b, c, d, along the edges the rule being that to the edge $\{a, b\}$ one attaches the two equal values $cdba, dcab$ having a, b at the end, etc. Thus, the sum of the three values for three edges having a common vertex is zero (even permutations, with a fixed last entry, amount to cyclic permutations of the first three entries). Now two edges not sharing a vertex (e.g., $\{c, d\}$ and $\{a, b\}$) have the same attached value since $-2abcd = -abcd - badc$ equals the sum of the values attached to the remaining four edges.