MATH 7721, SPRING 2018

Homework #12, February 5

PROBLEMS

1. Let g and h be two Riemannian metrics on an oriented surface Σ. As we know (see January 13 in the day-by-day list of topics), g and h turn the tangent bundle $T\Sigma$ into two complex line bundles $\mathcal{E}_g$ and $\mathcal{E}_h$. Prove that $\mathcal{E}_g$ and $\mathcal{E}_h$ are isomorphic as complex vector bundles. (Hint below)

2. Show that the Kähler form Ω of an oriented Riemannian surface (Σ, g) coincides with its area form $\mathrm{d}g$. (Hint below)

3. Verify that the Ricci form ρ of an oriented Riemannian surface (Σ, g) equals $K\,\mathrm{d}g$, where K is the Gaussian curvature of g.

Hint. In Problem 1, choose A as in Problem 3 of Homework #11, and note that, being orientation-preserving ($\det A > 0$, due to positivity) and isometric, A is complex-linear (since the almost-complex structure is uniquely determined by the metric and orientation).

Hint. In Problem 2: $\Omega_x(u, J_x u) = 1$ for any $x \in \Sigma$ and any unit vector $u \in T_x\Sigma$ (as $\Omega = gJ$), while $[\mathrm{d}g]_x(u, J_x u) = 1$ since $u, J_x u$ then is a positive-oriented orthonormal basis of $T_x\Sigma$ (Problem 2 in Homework #1).