MATH 7721, SPRING 2018

Homework #15, February 12

PROBLEMS

1. Given a twice-covariant tensor field a on a manifold M and a bundle morphism $B : TM \to TM$, let aB be the twice-covariant tensor field characterized by $(aB)(v, w) = a(Bv, w)$ for all vector fields v, w. (This generalizes the definition of aJ in the case where $B = J$ is an almost-complex structure.) Verify that $\pounds_w(aB) = (\pounds_w a)B + a\pounds_w B$ if a, B and a vector field w are all smooth.

2. The *tensor product* of 1-forms ξ, η of 1-forms on a manifold M is the twice-covariant tensor field $a = \xi \otimes \eta$ given by $a(v, w) = \xi(v)\eta(w)$ for vector fields v, w. Show that $\pounds_w a = (\pounds_w \xi) \otimes \eta + \xi \otimes \pounds_w \eta$ if ξ, η and w are smooth.

3. Prove that $\pounds_w df = d\pounds_w f$ whenever f is a smooth function on a manifold.