MATH 7721, SPRING 2018

Homework #16,   February 14

PROBLEMS

1. Verify that $\Psi(ZZ') = (\Psi Z)Z' + Z(\Psi Z')$ for any smooth vector fields w, u and tensor fields Z, Z' on a manifold, where $\Psi = \mathcal{L}_{[w,u]} - \mathcal{L}_w\mathcal{L}_u - \mathcal{L}_u\mathcal{L}_w$.

2. Prove that $\mathfrak{h}(M)$, for any Kähler manifold (M, g), is actually a complex Lie algebra, by establishing complex-bilinearity of the Lie bracket in $\mathfrak{h}(M)$.

3. Show that, on a compact oriented Riemannian manifold whose Ricci tensor is negative-semidefinite at each point, $\mathrm{Ker}\, D$ consists precisely of all parallel vector fields. (Hint below)

Hint. In Problem 3, use formula (16.9) in the day-by-day list of topics.