MATH 7721, SPRING 2018

Homework #17, February 16

PROBLEMS

1. Show that, on a compact oriented Riemannian manifold whose Ricci tensor is negative-semidefinite at each point, a smooth vector field is a Killing field if an only if it is parallel. (Hint below)

2. Verify that $\dim \mathfrak{i}(M,g) \leq n$ for any compact oriented n-dimensional Riemannian manifold (M,g), the Ricci of which is negative-semidefinite at every point.

3. Give an example of a compact oriented n-dimensional Riemannian manifold (M,g) with an open submanifold U such that (U,g) admits a Killing field not extendable to a Killing field on M. (Hint below)

Hint. In Problem 1, use formula (17.6) in the day-by-day list of topics.

Hint. In Problem 3, let (M,g) be a flat n-dimensional torus, $n \geq 2$ (that is, the Riemannian product of n circles), with U chosen to be isometric to an open submanifold of $\mathbf{R}^n$. Then, in view of Problem 1,

$$\dim \mathfrak{i}(M,g) \leq n < \frac{n(n+1)}{2} \leq \dim \mathfrak{i}(U,g),$$

where $n(n+1)/2$ is the dimension of the space of affine (linear plus constant) vector fields on $\mathbf{R}^n$ having a skew-adjoint linear part.