MATH 7721, SPRING 2018

Homework #19, February 21

PROBLEMS

1. Verify that equality (19.5) in the day-by-day list of topics holds for any two Killing fields u, v on a Riemannian manifold (M, g).

1. Given a Riemannian manifold (M, g), let $\mathcal{E}$ be the vector subbundle of $\mathrm{Hom}(TM.TM)$ whose sections are bundle morphisms $A : TM \to TM$, skew-adjoint at every point. Prove that the formula

$$\overline{\nabla}_w(u, A) = (\nabla_w u - Au, \, \nabla_w A - R(u, w))$$

defines a connection $\overline{\nabla}$ in the direct-sum vector bundle $TM \oplus \mathcal{E}$ and that the assignment $u \mapsto (u, \nabla u)$ is a linear isomorphism between $\mathfrak{i}(M, g)$ and the space of all $\overline{\nabla}$-parallel sections (u, A) of $TM \oplus \mathcal{E}$. (Hint below)

3. Show that, in a compact Lie algebra, all Lie subalgebras are compact, and all two-dimensional Lie subalgebras are Abelian. (Hint below)

Hint. In Problem 2, use formula (19.6) in the day-by-day list of topics.

Hint. In Problem 3, the first part claim is obvious. For the second: a two-dimensional non-Abelian Lie algebra has a basis u, w with $[u, w] = w$, so that $\mathrm{Ad}\, u$ cannot be skew-adjoint relative to any Euclidean inner product.