MATH 7721, SPRING 2018

Homework #21, February 26

PROBLEMS

1. Given a finite-dimensional complex vector space V with a Hermitian inner product $\langle\,,\rangle_{\mathbf{C}}$, and a complex-linear endomorphism A of V, show that the $\langle\,,\rangle_{\mathbf{C}}$-adjoint of A coincides with its $\langle\,,\rangle$-adjoint, for the Euclidean inner product $\langle\,,\rangle = \mathrm{Re}\,\langle\,,\rangle_{\mathbf{C}}$ in the underlying real vector space of V. (Hint below)

2. For $V, A, \langle\,,\rangle_{\mathbf{C}}$ and $\langle\,,\rangle$ as in Problem 1, verify that $\langle\,,\rangle_{\mathbf{C}}$-self-adjointness (or, $\langle\,,\rangle_{\mathbf{C}}$-skew-adjointness) of A is equivalent to its $\langle\,,\rangle$-self-adjointness (or, respectively, $\langle\,,\rangle$-skew-adjointness).

3. Prove that $\dim_{\mathbf{R}}\mathfrak{so}(V) = n(n-1)/2$ and $\dim_{\mathbf{C}}\mathfrak{u}(V) = m^2$ for the Lie algebra $\mathfrak{so}(V)$ (or, $\mathfrak{u}(V)$) of all real-linear (or, complex-linear) skew-adjoint endomorphisms of an n-dimensional real (or, m-dimensional complex) vector space V.

Hint. In Problem 1, apply Re to the relation $\langle Av, w\rangle_{\mathbf{C}} = \langle v, Bw\rangle_{\mathbf{C}}$, uniquely characterizing the $\langle\,,\rangle_{\mathbf{C}}$-adjoint B of A.