MATH 7721, SPRING 2018

Homework #24, March 5

PROBLEMS

1. Verify that $\delta(J\nabla f) = 0$ for any smooth function f on a Kähler manifold.

2. As a (very narrow) special case of the Hodge decomposition theorem, every smooth differential 1-form has an exact-coexact decomposition, in the sense of being the sum of an exact 1-form and one with zero divergence. Assuming compactness of M in addition to the other hypotheses required in (23.7), interpret equality (23.7) in the day-by-day list of topics, for any $v \in \mathfrak{h}(M)$, as an exact-coexact decomposition of the 1-form $-2\lambda g(v, \cdot)$.

3. We call two Riemannian metrics *homothetic* if some diffeomorphisms between the underlying manifold sends one metric onto a constant multiple of the other. Let a time-dependent Riemannian metric g on a manifold M be a *Ricci-flow trajectory*, in the sense that $\dot{g} = -2\mathrm{r}$. Prove that the following three conditions are equivalent:

(a) g satisfies the Ricci-soliton equation $\pounds_w g + \mathrm{r} = \lambda g$ at every time t, with some time-dependent vector field w and some time-dependent constant λ,

(b) g satisfies the Ricci-soliton equation $\pounds_w g + \mathrm{r} = \lambda g$ at some time t, with some fixed vector field w and some fixed constant λ,

(c) the metrics corresponding to all times t are mutually homothetic.

(Hint below)

Hint. In Problem 3,