

MATH 7721, SPRING 2018

Homework #3, January 12

PROBLEMS

1. Verify that an almost-complex submanifold of an almost-Kähler manifold becomes an almost-Kähler manifold when endowed with the submanifold metric.
2. Let us define a *Kähler connection* on an almost-complex manifold M to be any torsion-free connection ∇ in TM such that $\nabla J = 0$. If a compact almost-complex manifold admits a Kähler connection, does it also have to admit a Kähler metric? (Hint below)
3. Generalize Problem 1(c) in Homework #2 to the case of Kähler connections.

Hint. In Problem 2, the answer is ‘no’: Hopf manifolds carry flat Kähler connections, projected from $\mathbf{C}^m \setminus \{0\}$.