MATH 7721, SPRING 2018

Homework #6, January 22

PROBLEMS

1. Prove that $R(u,v)Jw = J[R(u,v)w]$ for the curvature tensor R of any Kähler manifold (M,g) and any vector fields u,v,w tangent to M. (Hint below)

2. For $(M,g), u, v$ and R as in Problem 1, show that $R(Ju,Jv) = R(u,v)$, where $R(u,v)$ is treated as a vector-bundle morphism $TM \to TM$. (Hint below)

3. For vector fields u,v tangent to a Riemannian manifold (M,g), we may treat $u \wedge v$ as a vector-bundle morphism $TM \to TM$, sending any vector field w to $g(u,w)v - g(v,w)u$. Verify that $R(u,v) = K u \wedge v$ in any Riemannian surface (M,g) with the Gaussian curvature K.

Hint. In Problem 1, note that R is the curvature tensor of a connection in a *complex* vector bundle.

Hint. In Problem 2, note that $R(u,v,Ju',Jv') = R(u,v,u',v')$, and use symmetries of R.