MATH 7721, SPRING 2018

Homework #9, January 29

PROBLEMS

1. Prove that, for the operator $i\partial\overline{\partial}$ induced by an almost-complex structure J on a manifold M, any torsionfree connection ∇ in TM, any smooth function f on M, and arbitrary vector fields u, v, one has

$$2[i\partial\overline{\partial}f](Ju,v) = -[\nabla df](Ju,Jv) - [\nabla df](v,u) - d_w f,$$

where $w = [\nabla_{Ju}J]v + J[\nabla_v J]u$. (Hint below)

2. Given a twice-covariant tensor field a on an almost-complex manifold M, verify that

(i) a is Hermitian if and only if a is symmetric and aJ is skew-symmetric,
(ii) a is skew-Hermitian if and only if a is skew-symmetric and aJ is symmetric.

Hint. In Problem 1, use formulae (3.2) and (1.23.a) in [KG] (the notes on Kähler geometry, https://people.math.osu.edu/derdzinski.1/courses/7721/kg.pdf):

$$2i\partial\overline{\partial}f = -d[(df)J], \qquad (d\xi)(u,v) = d_u[\xi(v)] - d_v[\xi(u)] - \xi([u,v]).$$