as in (8.27)). This obviously applies to the meson singlet described by $\mathbb{C}\cdot\mathrm{Id}_{V_n}$ as well.

Attempts to exhibit these mixed contents by selecting the K_α on the basis of naturality and simplicity always lead to results *depending on* n (cf. (viii)). This indicates that reality is more complicated and involves mixing among the quark-antiquark pairs $\mathrm{u\bar{u}}, \dots, \mathrm{t\bar{t}}$, as well as *mixing between specific multiplets* (see (viii)), the precise form of which is related to quark dynamics.

(viii)

Every $\mathrm{SU}(n)$ multiplet of hadrons or quarks is contained in a unique $\mathrm{SU}(n+1)$ multiplet. In fact, this is evident for quarks (§8.3). Thus, identifying the line summands L_j in (8.27) with quark flavors, we may regard V_n (along with its inner product) as a subspace of V_{n+1}. The summands of (8.19), (8.20) with $V = V_n$ then are contained in the corresponding summands for V_{n+1}, since the projection formulae in (i), (ii) of Remark 8.2.8 are dimension-independent. For the same reason, each line in these summands for V_n, standing for an individual baryon (see (v)) is one of the analogous lines for V_{n+1}, which establishes our claim for baryon multiplets of types (ii)c),d). On the other hand, $\mathfrak{sl}(V_n) \subset \mathfrak{sl}(V_{n+1})$ under the trivial-extension relation $\mathrm{End}\, V_n \subset \mathrm{End}\, V_{n+1}$ corresponding to the tensor-product inclusion $V_n\overline{V}_n \subset V_{n+1}\overline{V}_{n+1}$ (cf. (8.22)). Consequently, if the lines $K_\alpha \subset \mathfrak{sl}(V_n)$ representing the $n-1$ strictly neutral mesons contained in the $\mathrm{SU}(n)$ multiplet of (iii)d) are chosen so as to yield mixed quark contents that do not depend on n (see (vii)), these mesons (along with the other multiplet members, described by $L_j\overline{L}_k$ with $j \neq k$, cf. (v)) will explicitly become a part of an $\mathrm{SU}(n+1)$ multiplet. Thus, the above assertion is valid for all hadrons, the cases of $\mathrm{SU}(n)$ singlets and n-tuplets being obvious.

Formally, however, the singlet in (iii)d) leads to some problems, since the $\mathbb{C}\cdot\mathrm{Id}$ summand of (8.22) with $V = V_n$ is contained in neither such summand for V_{n+1}. This suggests that, at least in some cases, instead of belonging to just one of the two $\mathrm{SU}(n)$ multiplets in (iii)d), strictly neutral mesons may rather be *mixtures* involving “ideal” objects from *both* multiplets.

(ix)

The descriptions (ii), (iii) of $\mathrm{SU}(n)$ multiplets of hadrons also determine the distribution of values assumed by the quantum numbers Q, B, I_3, Y, S, C, b, t (§2.14) in an $\mathrm{SU}(n)$ multiplet with any prescribed quark content modulo $\mathrm{SU}(n)$ and quark composition (cf. Remark 8.3.1). In fact, they all vanish for strictly neutral mesons, while for other hadrons their values can be obtained, by additivity, from the collection of quark contents occurring within the multiplet (§8.1, and (v) above).

We can now proceed to identify $\mathrm{SU}(n)$ *multiplets in nature* by grouping the known hadron species into finite sets with pairwise disjoint mass ranges (Remark 8.3.5) for any given values of spin and parity (determined by the quark composition, and hence constant on each set), which