9 Hadrons and the Weak Interaction*

The main topic discussed in this chapter is the geometric structure of the *Glashow-Illiopoulos-Maiani model,* which is an extension of the electroweak theory to quarks, and through them, to hadrons. Additional complexities involved in such an extension include the phenomenon of *weak mixing* of quark flavors, the extent of which is characterized by an angular parameter introduced by Cabibbo and later generalized by Kobayashi and Maskawa.

9.1 Natural Subspaces of Direct Sums

Let $V \oplus \ldots \oplus V$ be the direct sum of n copies, $n \geq 2$, of a finite dimensional complex vector space V, possibly endowed with some additional geometric structure. A subspace V' of $V \oplus \ldots \oplus V$ is called *natural* if it is obtained by a general (functorial) construction involving the geometry of V. Obvious examples of natural subspaces are

$$V'_{a_1,\ldots,a_n} = \{(a_1 v, \ldots, a_n v) \in V \oplus \ldots \oplus V : v \in V\} \tag{9.1}$$

with fixed $a_j \in \mathbb{C}$ satisfying $\sum_j |a_j| > 0$, which may (and always will) be chosen so that $\sum_j |a_j|^2 = 1$. (About "universality" of the examples (9.1), see Remark 9.1.1 below.) A collection

$$V'_{a_{j1},\ldots,a_{jn}}, \quad j = 1, \ldots, n,$$

of n such subspaces is said to form an *orthogonal n-tuple* in $V \oplus \ldots \oplus V$ if the matrix $a = [a_{jk}]$ is unitary. This is obviously the case if and only if the subspaces in question are pairwise orthogonal with respect to an inner product $\langle , \rangle$ in $V \oplus \ldots \oplus V$, definite or not, obtained by extending any Hermitian inner product in V so as to make the summands orthogonal. If $n = 2$, all orthogonal pairs $V', V'' \subset V \oplus V$ are, clearly, of the form

$$V' = V'_{a_1,a_2}, \quad V'' = V'_{\bar{a}_2,-\bar{a}_1}. \tag{9.2}$$

For $n = 2$, we define the *Cabibbo angle* θ_C corresponding to the natural subspace $V' = V'_{a_1,a_2}$ of $V \oplus V$ (or, to the orthogonal pair (9.2)) by

$$\tan\theta_\mathrm{C} = |a_2/a_1|, \quad 0 \leq \theta_\mathrm{C} \leq \tfrac{\pi}{2} \tag{9.3}$$