Errata for

Integral, Probability, and Fractal Measures

March 3, 2022 by G. A. Edgar

Page vii Line 14. Delete every $n \in \mathbb{N}$ and **Page 10 Line 18. After** Thomson **Add** [258],[259] **Page 10 Line -6.** After $\overline{B}_r(x) \cap \overline{B}_{r'}(x')$ Add $= \emptyset$ **Page 20 Line 5.** Replace $\mathcal{P}^{s}(A) = \overline{\mathcal{P}}^{s}(E)$ by $\overline{\mathcal{P}}^{s}(A) = \overline{\mathcal{P}}^{s}(E)$ **Page 20 Line 6. Replace** Let E_n be an increasing sequence **by** Let $E_n \nearrow E$ be a sequence. Page 21 Line 20. Replace Vitali cover by fine cover Page 33 Line -12. After measurable sets. **add** See Exercise 10.51 in [250], dated 1965. **Page 34 Line -13. Replace** It is a metric outer meeasure. by It is not a metric outer measure (Bilel Selmi, manuscript). Page 36 Line -4. Replace | by | | $y \in S$ $y \in A$ Page 38 Line 6. Replace $t_1 = 0$ by $t_1 = a$ Page 38 Line -11. Replace $N_{\varepsilon/2}(C)$ by $N_{\varepsilon}(C)$ **Page 45 Line -6. Replace** of course) that by of course) that if E and F are nonempty separable metric spaces, then Page 46 Line -3. Replace any any by any Page 50 Line -4. Replace centered Vitali cover by fine cover Page 51 Line 12. Replace $\overline{D}^{s}(x)$ by $\overline{D}^{s}_{\mathcal{M}}(x)$ Page 51 Line -2. Replace $C^{s}(F)$ by $C^{s}(F)$ **Page 53 Line 3. Replace** sup $\overline{D}^{s}_{\mathcal{M}}(x)$ by $\inf \overline{D}^{s}_{\mathcal{M}}(x)$ **Page 58 Line -6. Replace** $\mathcal{H}^s_{\mathcal{B}}(F) \leq pq^s \mathcal{H}^s_{\mathcal{A}}(F)$ by $\overline{\mathcal{H}}^s_{\mathcal{B}}(F) \leq pq^s \overline{\mathcal{H}}^s_{\mathcal{A}}(F)$ Page 59 Line 5. Replace $\mathcal{H}^s_{\mathcal{B}}(F) \leq pq^s \mathcal{H}^s_{\mathcal{A}}(F)$ by $\overline{\mathcal{H}}^s_{\mathcal{B}}(F) \leq pq^s \overline{\mathcal{H}}^s_{\mathcal{A}}(F)$ **Page 59 Line 12. Replace** $\mathcal{H}^{s}_{\mathcal{A}_{2}}(h[F]) \leq pq^{s}\mathcal{H}^{s}_{\mathcal{A}_{1}}(F)$ by $\overline{\mathcal{H}}^s_{\mathcal{A}_2}(h[F]) \leq pq^s \overline{\mathcal{H}}^s_{\mathcal{A}_1}(F)$ Page 59 Line 15. Replace $\mathcal{H}^{s}_{\mathcal{A}_{1}}(h^{-1}[E]) \leq pq^{s}\mathcal{H}^{s}_{\mathcal{A}_{2}}(E)$ by $\overline{\mathcal{H}}^s_{\mathcal{A}_1}(h^{-1}[E]) \le pq^s \overline{\mathcal{H}}^s_{\mathcal{A}_2}(E)$ **Page 61 Line 19.** Replace $K \subseteq S$ by $K \subseteq F$ **Page 62 Line 11. Replace** $\mathcal{H}^{s}_{\delta}(E) > \alpha$ by $\mathcal{H}^{s}_{2\delta}(E) > \alpha$. Choose a sequence δ_i so that $2\delta > \delta_1 > \delta_2 > \cdots$ and $\delta_i \to \delta$. **Page 62 Line 12. Replace** there is k_1 by by (1.2.8) there is k_1 Page 62 Line 13.Replace \mathcal{H}^s_{δ} by $\mathcal{H}^s_{\delta_1}$ Page 62 Line 15.Replace \mathcal{H}^s_{δ} by $\mathcal{H}^s_{\delta_2}$ Page 62 Line 17. Replace \mathcal{H}^s_{δ} by $\mathcal{H}^{s^{-}}_{\delta_i}$ **Page 62 Line 20.** After compact sets add and $\mathcal{H}^{s}_{\delta}(A_{j}) > \alpha$ **Page 64 Line 5.** Add [Answer: No in general, yes in \mathbb{R}^d : A. Schechter, 1998.]

Page 67. Insert at the top

(1.5.12): The Strong Vitali Property is not used in (a), so the inequality

$$\mathcal{P}^{s}(E) \inf_{x \in E} \underline{D}^{s}_{\mathcal{M}}(x) \le \mathcal{M}(E)$$

does not need this assumption. Without the Strong Vitali Property we may prove a modified form of the second inequality:

$$\mathcal{M}(E) \le 3^s \mathcal{P}^s(E) \sup_{x \in E} \underline{D}^s_{\mathcal{M}}(x).$$

To do this, use (1.3.1) in place of the Strong Vitali Property. See Cutler [52]. Mattila & Mauldin (*Math. Proc. Camb. Phil. Soc.* **121** (1997) 81–100) provide an example in Hilbert space where the inequality of (1.5.11) is false with constant 1.

Page 88 Line -9. After the set E. add Remember (1.4.20) and (1.5.10).

Page 96 Line 2. replace $\Phi(\mathbb{1}_U)$ by $\Phi(1)$

- Page 106. Everywhere, replace $\varepsilon/5$ by $\varepsilon/7$ and $\varepsilon/(5\gamma)$ by $\varepsilon/(7\gamma)$
- Page 106 Line 19. Replace $\varepsilon/10$ by $\varepsilon/14$
- Page 106 Line -8. After $p\eta < 1$ add $, \eta p\gamma < 1,$
- Page 106 Line -4. Replace < by \leq

Page 107. Replace first two lines by

$$\leq (-\gamma)\mathcal{M}(E_0) + \sum_{i=1}^p \left(h(x_i) - \frac{\varepsilon}{7}\right)\mathcal{M}(E_i) \\ + 2\gamma \frac{\varepsilon}{7\gamma} + \gamma\eta + 2\frac{\varepsilon}{7}\sum_{i=1}^p \mathcal{M}(E_i) + \eta p\gamma + \eta p\frac{\varepsilon}{7}$$

Page 107 Line -11. Replace converges to E_0 by converges narrowly to E_0

- Page 108 Line -5. Replace $dist(x, U_k)$ by $dist(x, S \setminus U_k)$
- Page 108 Line -4. Replace $(2/\varepsilon)f_k$ by $(\varepsilon/2)f_k$

Page 108 Line -3. After $\varepsilon < 1$). add Recall (p. 105) V_1 is the set of functions with Lipschitz constant 1 and bounded by 1 used to define the metric ρ_1 .

- Page 109 Line 9. Replace K is by K is
- **Page 118 Line 14. Replace** if s < b if s > b if s > b if $s \ge b$ if $s \le b$
- Page 121 Line 13. Replace $F \subseteq A$ by $F \subseteq E$
- Page 127 Line 15.

Delete Suppose in addition that \mathcal{M} has the Strong Vitali Property. Then **Page 127 Line -4. Replace the parenthesis by**

(the variant on page 67 that does not require the Strong Vitali Property) Page 132. Figure 3.3.21. Label T_1 is missing.

- Page 136 Line -1. Replace $F_1(0,1)$ by $F_1(1,1)$ and $F_2(0,1)$ by $F_2(1,1)$
- Page 137 Line 1. Replace $F_3(0,1)$ by $F_3(1,1)$ and $F_4(0,1)$ by $F_4(1,1)$

Page 174 Line 20. Replace the standard normal and Cauchy by certain normal and Cauchy

- **Page 185 Line 7. Replace** for all $t \in \mathbb{R}$ by for D_X -almost all $t \in \mathbb{R}$
- Page 186 Line -5. Replace the third σ' by σ
- Page 191 Line -7. Replace \leq by \geq

Page 193. Replace footnote ¹⁰ by

TIMING TOAST Grook on how to char for yourself

There's an art of knowing when. Never try to guess. Toast until it smokes and then twenty seconds less.

Piet Hein, Grooks 2, p. 23

Page 195 Line 16. Replace most likely by more likely

Page 197 Line 16. After dyadic subintervals add $[j/2^k, (j+1)/2^k]$

- **Page 202.** Add (4.3.18): the converse is not true
- **Page 211 Line -15.** Replace all U by all $e \in E$ and $f_{e'}U$ by $f_{e'}[U]$
- **Page 211 Line -14.** Replace $e \neq s' \in E$ by $e \neq e' \in E$
- **Page 213 Line 19. Replace** Show that Lemma 5.2.4 remains **by** Does Lemma 5.2.4 remain
- Page 216 Line 1. After triangle T add (p. 214)
- **Page 228 Line -2. Replace** n(2i) by n(2i, k+1)
- **Page 229 Line 1. Replace** n(2i) by n(2i, k+1)
- Page 229 Line 9. Replace I_i by I_1
- Page 231. Figure 5.5.1. The second and third graphs are interchanged.
- **Page 235 Line -3. Replace** denominator k+1 by denominator 2^{k+1}
- **Page 238 Line 17. Replace** Define $\mathbf{X}(t) = \mathbf{by}$ Define $\mathbf{X}(x) =$
- **Page 239 Line -5. Replace** certain constants C_1 and C_2 by a certain constant C_1
- Page 239 Line -2. Replace $C_2(t-s)^{\gamma}$ by $C_1(t-s)^{\gamma}$
- Page 240 Line 11. Replace by q factor by by a factor
- Page 247 Line -9. Replace $k^2/2^{k-1}$ by $\frac{k^2}{2^{k-1}}$
- Page 248 Line 13. After each jump add and their limit points
- Page 251 Line -4. Replace X'(t) = X(t+p) X(p)by X'(t) = X(t+p) - X(p)

Page 254 Line 7. Replace Brow- nian by Brown- ian Page 259 Line 14. Replace $p_e = r_e^{\beta'(q)}$ by $p_e = r_e^{-\beta'(q)}$

- Page 264 Line -9. Replace $(2((1/4)^s + (1/8)^s))^k$ by $((1/4)^s + (1/8)^s)^k$
- Page 264 Line -7. Replace 0.8232 by 0.4057
- Page 264 Line -6. Replace 0.5652 by $0.7549 \mbox{ and } 1/2$ by 1
- Page 264 Line -5. Replace display by

$$\lambda = \frac{\sqrt[3]{100 + 12\sqrt{69}}}{6} + \frac{\sqrt[3]{100 - 12\sqrt{69}}}{6} - \frac{1}{3}$$

Page 265 Line 6. Replace paragraph by (5.4.12) The number of descendents eventually reaches zero (so that $K = \emptyset$) when $\Phi(0) = 1$ except in a "deterministic" situation where there is exactly one descendent with probability 1. The number of descendents after n steps is a nonnegative martingale, so it converges; then argue that the limit must be 0 almost surely.

Page 283. Before Haase add grook, 193

and before Henstock add Hein, Piet, 193

Page 284. Replace multifractal decomposition, 25

by multifractal decomposition, 257