Errata for November 20, 2011
Stopping Times and Directed Processes by Edgar & Sucheston

Page x Line -6. Replace (5.3.30 and 5.3.34) **by** (5.4.13 and 5.4.17)
Page xi Line 7. Replace extended **by** applied
Page 8 Line 1. After If **add** $\sigma, \tau \in \Sigma$ and
Page 14 Line 12. Replace 4.2.8 and 4.2.11 **by** 4.2.7 and 4.2.10
Page 21. Replace the first paragraph of the proof of (1.4.3) by
Proof. (i) Let (X_t) be a submartingale. We first prove that $\mathbf{E}^{\mathcal{F}_\sigma}[X_\tau] \geq X_\sigma$ for $\sigma, \tau \in \Sigma^{\mathrm{O}}$, $\sigma \leq \tau$. By the localization theorem 1.4.2, it suffices to show that, for each s, we have $\mathbf{E}^{\mathcal{F}_s}[X_\tau] \geq X_s$ on the set $\{\sigma = s\}$. Fix a value s of σ. We proceed by induction on the number of values τ takes on the set $\{\sigma = s\}$. If τ takes only one value t on $\{\sigma = s\}$, then $\mathbf{E}^{\mathcal{F}_s}[X_\tau] = \mathbf{E}^{\mathcal{F}_s}[X_t] \geq X_s$ on $\{\sigma = s\}$ since (X_t) is a submartingale. For the inductive step, suppose τ takes values $t_1 < t_2 < \cdots < t_n$ on $\{\sigma = s\}$, where $n \geq 2$. Since $\tau \geq \sigma$, we have $t_1 \geq s$. Define $\tau' \in \Sigma^{\mathrm{O}}$ by
(continue as in the book)
Page 25 Line -1. At the bottom of the page add
The process (Z_t) in (ii) is known as an *amart potential*.
Page 30 Line -11. Replace weak truncated **by** weak L_1
Page 30 Line -10. Replace $[X_n]$ **by** $[X_i]$
Page 80 Line 3. Replace bounded **by** integrable
After Poussin **add** (2.3.5)
Page 88 Line 2. Replace $\mu\{|f| \geq \lambda\}$ **by** $\mu\{|g| \geq \lambda\}$
Page 96 Line 10. Replace (Improved constants.) **by** (Best constants.)
Page 97 Line -11. After $\leq$ **add** $1/\lambda$
Page 113 Line -9. Replace (4.2.18) **by** (4.2.17)
Page 113 Line -8. Replace (4.2.8) **by** (4.2.7)
Page 113 Line -4. Replace (4.2.19) **by** (4.2.18)
Page 118 Line 3. Replace (4.1.7a) **by** (4.1.6a)
Page 119 Line 10. Replace $\widehat{A} \subseteq A$ **by** $A \subseteq \widehat{A}$
Page 119 Line 11. Replace $B \subseteq A$ **by** $B \subseteq \widehat{A}$
Page 125 Line 9. Replace submartingale **by** supermartingale
Page 137 Line -7. Replace (4.2.11) **by** (4.2.10)
Page 138 Line -7. Replace (4.2.9) **by** (4.2.8)
Page 140 Line -12. Replace (8.4.1) **by** (8.1.4)
Page 140 Line -7. Replace (4.2.11) **by** (4.2.10)
Page 140 Line -6. Replace (4.2.8) **by** (4.2.7)
Page 141 Line -3. Replace $(X_t)_{t \in J}$ **by** $(\mathcal{F}_t)_{t \in J}$
Page 141 Line -1. Replace (4.2.8) **by** (4.2.7)
Page 146 Line 17. Add (as in (a) $\Longrightarrow$ (b) of 4.2.3.)
Page 156 Line 13. Replace This disadvantage **by** The disadvantage
Page 157 Line -3. After (X_t) **add** is
Page 158 Line -14. Replace $p = \infty$ **by** $p = 1$ **and** (4.2.12) **by** (4.2.13)
Page 169 Line -6. Replace (A_t) **by** $(\mathcal{F}_t)$

Page 180 Line -1. After in **add** real **and replace** (5.1.22) **by** (2.3.11)
Page 200 Line 12. Replace (5.1.9a) **by** (5.1.10a)
Page 199. Replace (5.3.6) by

(5.3.6) Proposition. *Let $(X_n)_{n\in\mathbb{N}}$ be a process with values in a Banach space E. If X_n converges a.s. in norm to X and $\{X_n\}$ is uniformly absolutely continuous, then $X_n - X$ converges to 0 in Bochner norm and in Pettis norm.*

Proof. We have $\|X_n - X\| \to 0$ a.s., so $\|X_n\| \to \|X\|$ a.s. Let $\varepsilon > 0$. There is $\delta > 0$ so that for any event A with $\mathbf{P}(A) < \delta$ we have $\mathbf{E}[\|X_n\|\mathbf{1}_A] < \varepsilon$ for all n. By Fatou's Lemma, also $\mathbf{E}[\|X\|\mathbf{1}_A] < \varepsilon$ so that $\mathbf{E}[\|X_n - X\|\mathbf{1}_A] < 2\varepsilon$. Write $A_n = \{\,\omega \in \Omega : \|X_n(\omega) - X(\omega)\| > \varepsilon\,\}$. Now by Egorov's Theorem, there is $N \in \mathbb{N}$ so that $\mathbf{P}(A_n) < \delta$ for all $n \geq N$. Therefore for all $n \geq N$,

$$\mathbf{E}[\|X_n - X\|] = \mathbf{E}[\|X_n - X\|\mathbf{1}_{A_n}] + \mathbf{E}\left[\|X_n - X\|\mathbf{1}_{\Omega\setminus A_n}\right] \leq 2\varepsilon + \varepsilon = 3\varepsilon.$$

Therefore, $\|X_n - X\|_{L_1} \to 0$. Since the Bochner norm is $\geq$ the Pettis norm, we also have $\|X_n - X\|_P \to 0$. ■

Page 200 Line 9. Delete and in Pettis norm
Page 201 Line 5. Delete and hence in Pettis norm
Page 206 Line 21. Replace these are **by** but $\langle X_n, x^*\rangle$ are
Page 207 Line 2. Replace bounded **by** closed bounded convex
Page 214 Line -9. Replace X_n **by** X_t
Page 218 Line 6. Replace (5.3.10) **by** (5.3.13)
Page 220 Line -11. Replace $x_0 = y_0 + z_0$ **by** $x_0 = (1/2)(y_0 + z_0)$
Page 223 Line 15. Replace finite intersection property
by countable intersection property
Page 225 Line 2. Replace $S(C, x^*, 2\alpha)$ **by** $S(C, x^*, \alpha)$
Page 225 Line 3. Replace this slice **by** the slice $S(C, x^*, \alpha/2)$
Page 228. Figure (5.4.15) Replace $y^* = \eta$ **by** $y^* = \gamma$
Page 243. In the diagram, add an arrow from L_1 to E with label T.
Page 302 Line 1. Replace Let U be a bounded open set.
by Let $U \subseteq \mathbb{R}^d$ be an open set, and let $\mathcal{C}$ be a substantial Vitali cover of U consisting of nonempty bounded open sets with boundaries of measure zero.
Page 305. Figure (7.2.6) Replace I_1 **by** E_1**, replace** I_2 **by** E_2
and replace I_N **by** E_N
Page 319 Line 12. Replace $(1/a)\Psi(v)$ **by** $\Psi(v/a)$
Page 321 Line 3. Replace $\mu(C \cap \bigcup \mathcal{A})$ **by** $\eta\mu(C \cap \bigcup \mathcal{A})$
Page 322 Line 17. Replace $\sum \mu(B_k) < \infty$
by $\sum \mu(B_k) = \int n_{\mathcal{A}}\, d\mu \leq \mu(C \cap \bigcup \mathcal{A})/(1-\eta) < \infty$
Page 335 Line 7. After diameter.
add By (7.4.2), $(\mathcal{E}_t, \delta)$ has small overflow.
Page 344 Line 14. Replace (8.4.6) **by** (8.6.4)
Page 354 Line 5. Replace $c = 1$ **by** $\|T\|_\infty \leq 1$

Page 372 Line -13. After functions.

add In the theorem below, an *exact dominant* is a dominant such that its integral is equal to the infimum of integrals taken over all dominants. In the Markovian case this definition coincides with that given on p. 366.

Page 373 Line 2. Replace $(2) \Longrightarrow (4)$ **by** $(2) \Longrightarrow (3)$

Page 373 Line 17. Replace $\left(1+\frac{1}{m}\right)$ **by** $\left(1-\frac{1}{m}\right)$

Page 373 Line 20. Replace π_2 **by** π_0

Page 373. Replace line -4 by

It follows that if η is finite, then $\delta = d\eta/d\mu$ is a dominant. We show that $\eta(\Omega) \leq \beta$, hence η is finite. Since T^{**} is a contraction, $\big(\eta_1 + \pi_1\big)(\Omega) = T^{**}\pi_0(\Omega) \leq \pi_0(\Omega)$. Hence

$$\big(\eta_0 + \eta_1\big)(\Omega) \leq \big(\eta_0 + \pi_0 - \pi_1\big)(\Omega) \leq \big(\eta_0 + \pi_0\big)(\Omega) = \psi_0(\Omega) \leq \beta.$$

We show that $\big(\eta_0 + \eta_1 + \cdots + \eta_n\big)(\Omega) \leq \beta$ for all n, hence $\eta(\Omega) \leq \beta$. The induction hypothesis is: $\big(\eta_0 + \eta_1 + \cdots + \eta_n\big)(\Omega) \leq \big(\eta_0 + \pi_0 - \pi_n\big)(\Omega)$. Since $\eta_{n+1} = M(T^{**}\pi_n)$ and $\pi_{n+1} = T^{**}\pi_n - \eta_{n+1}$, we have

$$\big(\eta_{n+1} + \pi_{n+1}\big)(\Omega) = T^{**}\pi_n(\Omega) \leq \pi_n(\Omega).$$

Therefore $\eta_{n+1}(\Omega) \leq \big(\pi_n - \pi_{n+1}\big)(\Omega)$. Now $\big(\eta_0 + \eta_1 + \cdots + \eta_n + \eta_{n+1}\big)(\Omega) \leq \big(\eta_0 + \pi_0 - \pi_n\big)(\Omega) + \big(\pi_n - \pi_{n+1}\big)(\Omega) = \big(\eta_0 + \pi_0 - \pi_{n+1}\big)(\Omega)$. This proves $(2) \Longrightarrow (3)$.

Page 373 Line -3. After $(3) \Longrightarrow (2)$ **add** and $(3) \Longrightarrow (4)$

Page 383 Line 4. Replace (9.4.5) **by** (9.4.4)

Page 383 Line 6. Replace Proposition (9.4.5)

by Proposition following (9.4.6).

Page 383 Line 10. Replace (9.4.8) **by** (9.4.7)

Page 393 Line -14. Replace L_1 **by** L_p **and** (2.1.14c) **by** (2.2.14c)

Page 393 Line -10. Replace (2.1.14c) **by** (2.2.14c)

Page 399 Line -5. Add Convergence in $L\log^{d-1} L$ follows from (2.3.22).

Page 404 Line -10. Replace (9.4.10(i)) **by** (9.4.11)

Page 405 Line 10. Replace (4.2.11) **by** (4.2.10)

Page 406 Line -8, -9. Replace 9.4.1 **by** 9.4.4 **(three times)**

Page 409. In reference Burkholder [1991] **replace** 1–665 **by** 1–66

Page 418. After Fava, N. A., 33, **add** 57,

Page 421. After affine fixed-point property, 223

add Akcoglu's theorem, multiparameter, 393

Page 422. After criterion of de La Vallée Poussin **replace** 71 **by** 72

Page 423. After Fava spaces, **add** 57,

Page 425. After Mourier theorem, **add** multiparameter,